CS3231 : Tutorial - 4

Rahul Jain

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1. Give contrext-free grammars for the following languages. Below $\Sigma = \{0, 1\}$.
   (a) $\{w|\ w$ contains at least three $1s\}$.
   (b) $\{w|\ w$ starts and ends with the same symbol $\}$.
   (c) $\{w|\ w$ the length of w is odd and its middle symbol is $0\}$.
   (d) $\{w|\ w$ is a palindrome$\}$.
   (e) $\{w|\ w$ has more ones than zeros $\}$.
   (f) The complement of the language $\{0^n 1^n|\ n \geq 0\}$.
   (g) $\{a^i b^j c^k \in \{a, b, c\}^*|\ i = j$ or $j = k$ where $i, j, k \geq 0\}$. Is your grammar ambiguous ? Why or why not ?

2. Show that the class of context free languages is closed under the regular operations, union, concatenation and star.

3. Convert the following CFG to Chomsky normal form.

$$A \to BAB \mid B \mid \varepsilon$$
$$B \to 00 \mid \varepsilon$$

4. Show that, if G is a CFG in Chomsky normal form, then for any string $\{w \in L(G)\}$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.

5. Use the languages $A = \{a^m b^n c^n|\ m, n \geq 0\}$ and $B = \{a^n b^n c^m|\ m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.

   Use the above and De-Morgan's law to show that the class of context-free languages is not closed under complementation.

6. Let C be a context free language and R be a regular language. Prove that the language $C \cap R$ is context free. Use this to show that the language $A = \{w \in \{a, b, c\}^*|\ w$ contains equal number of a's, b's and c's$\}$ is not a CFL.