ICP and FICP

[ICP] P. J. Besl and N. D. McKay. A method for registration of 3-D shapes. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 14(2):239–256, 1992.

[FICP] Phillips J. M., Liu R., and Tomasi C. Outlier robust ICP for minimizing fractional RMSD. In *Proc. of Int. Conf. on 3D Digital Imaging and Modeling* (2007), pp. 427–434.

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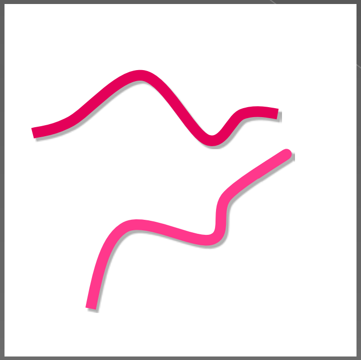
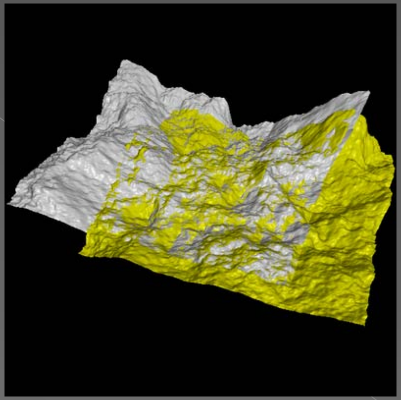
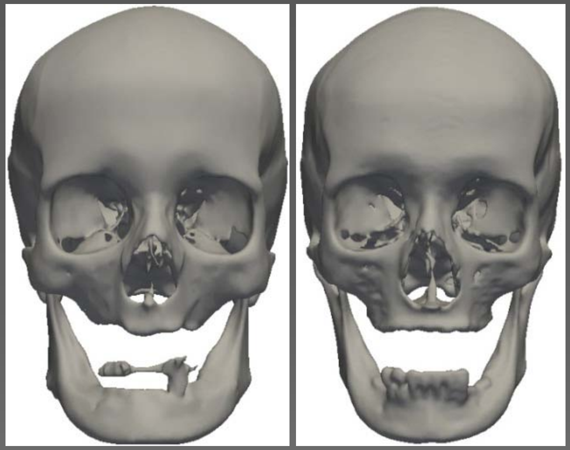
Align 3D data using rigid transformation

Points Lines Surfaces 3D objects

Iterative Closest Point (ICP)

- P. J. Besl and N. D. McKay. A method for registration of 3-D shapes. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 14(2):239–256, 1992.

Rigid transformation

- Rigid objects

Rotation

Translation

$$T(p) = sRp + t$$

Scaling

Known Correspondence

- Problem formulation:
  - **p_i** points on the model.
  - **q_i** corresponding points of p_i on the object.
  - Find the linear transformation **T** that minimizes the error **E**:

$$E = \frac{1}{N} \sum_{p_i \in M} d^2(T(p_i), q_i)$$

Number of points in M

Euclidean Distance

Step 1 translation

- Assume s and R are fixed then, the optimal t that minimizes:

$$E = \frac{1}{N} \sum_{p_i \in M} \|sRp_i + t - q_i\|^2$$

- should be:

$$t = \frac{1}{N} \sum_{p_i \in M} (q_i - sRp_i)$$

$$t = \overline{q_i} - sR\overline{p_i}$$

Step 2 scale

- Replace the t in the error function using the previous result we get:

$$E = \frac{1}{N} \sum_{p_i \in M} \|sRp'_i - q'_i\|^2$$

- Where:

$$p'_i = p_i - \overline{p_i}$$

$$q'_i = q_i - \overline{q_i}$$

Step 2 scale

- Expanding the error function

$$E = \frac{1}{N} \sum_{p_i \in M} \|sRp'_i - q'_i\|^2$$

$$E = \frac{1}{N} \sum_{p_i \in M} \|sRp'_i\|^2 - 2\frac{1}{N} \sum_{p_i \in M} (sRp'_i)q'_i + \frac{1}{N} \sum_{p_i \in M} \|q'_i\|^2$$

- Rewritten as:

$$E = \frac{1}{N}(s^2 S_p - 2sD + S_q)$$

Step 2 scale

$$E = \frac{1}{N}(s^2 S_p - 2sD + S_q)$$

- Minimized when:

$$s = \frac{D}{S_p}$$

Step 3 rotation

- Use quaternion representation of rotation **q**:
  - Rotation of a point r_i can be written as:

$$qr_iq^*$$

  - The problem becomes minimize:

$$\begin{aligned} f(q) &= \sum_{i=0}^{n-1} \|qr_iq^* - s_i\|^2 \\ &= \sum_{i=0}^{n-1} (\|r_i\|^2 \boxed{- 2\langle qr_iq^*, s_i\rangle} + \|s_i\|^2) \end{aligned}$$

Step 3 rotation

- Then we need to maximize:

$$f_1(q) = q^T \left(\sum_{i=0}^{n-1} \bar{\mathcal{R}}_i{}^T \mathcal{S}_i \right) q.$$

- Thus the optimal rotation **q** is the eigen vector corresponding to the largest eigen value of $\left(\sum_{i=0}^{n-1} \bar{\mathcal{R}}_i{}^T \mathcal{S}_i \right)$

Without correspondence

Without correspondence

- Make **educated guess.**
- When two objects are registered.
  - points are **close to** some points on the other object.
- Given a point $\mathbf{p_i}$ in M, the closest point $\mathbf{q_i}$ in **O** satisfies:

$$d(p_i, q_i) = \min_{q_j \in O} d(p_i, q_j)$$

Euclidean distance

ICP algorithm

- Repeat until convergence:
  - Find q_i of p_i.
  - Compute **T** that minimizes:

$$E = \sum_{p_i \in M} d^2(T(p_i), q_i)$$

  - Apply the **T** to all the p_i .

Converge to local minimum

smaller E

- Repeat until convergence:
  - Find q_i of p_i.
  - Compute **T** that minimizes:

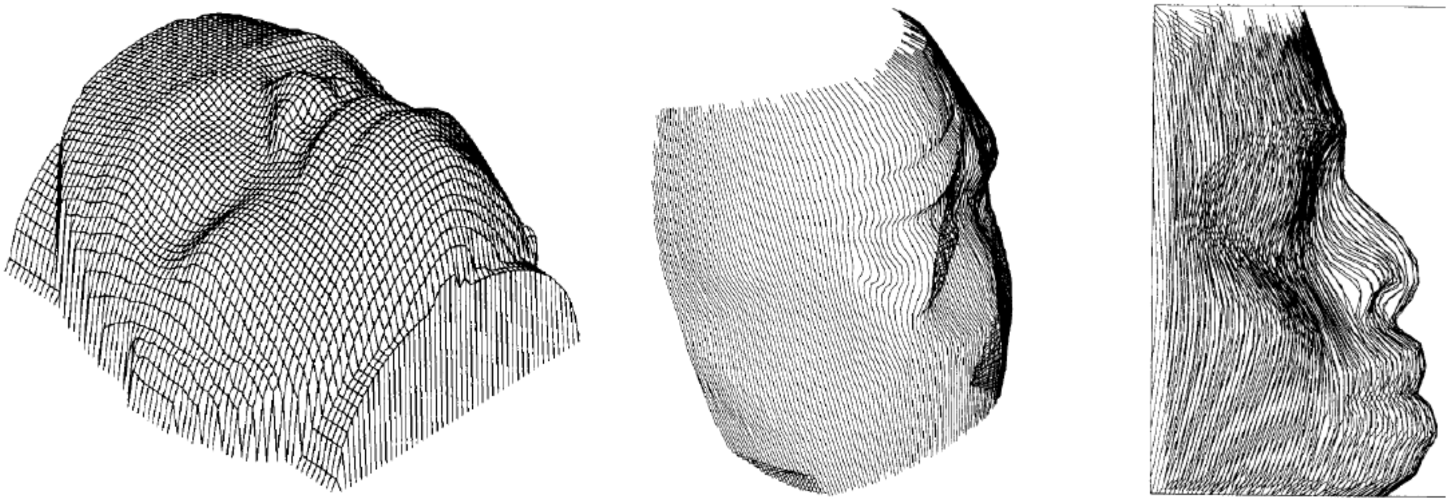
$$E = \sum_{p_i \in M} d^2(Tp_i, q_i)$$

smaller E

  - Apply the **T** to all the p_i .

Results

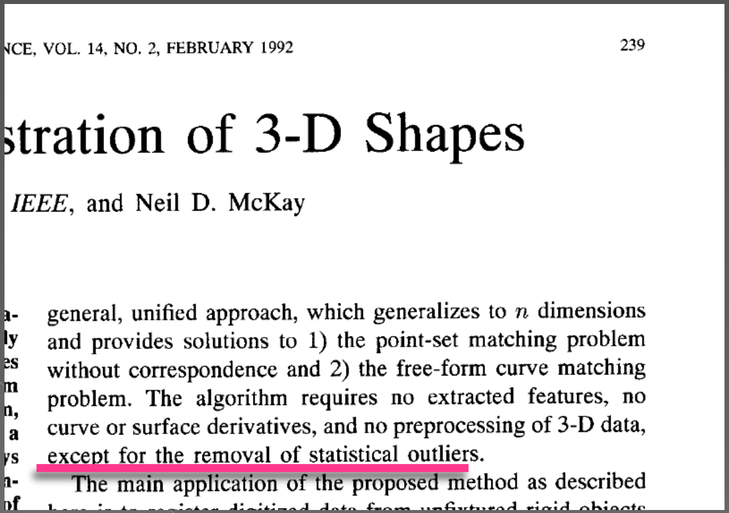
Example: From [BM92].



(a) model 1 (b) model 2 (c) registered models

Conclusion

- Various data representation
- Outlier sensitive
- Initialization sensitive



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stration of 3-D Shapes

IEEE, and Neil D. McKay

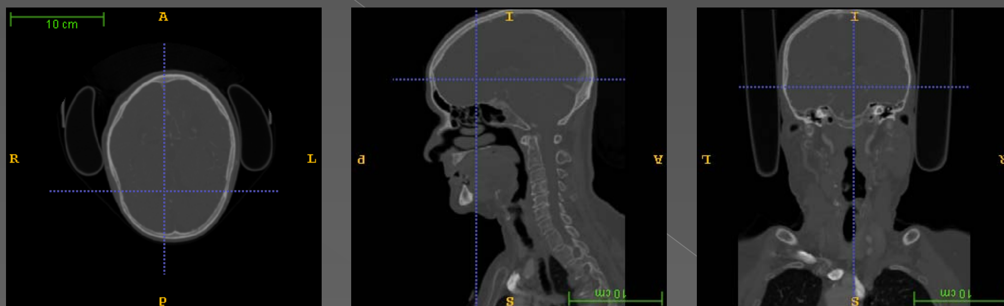
general, unified approach, which generalizes to n dimensions and provides solutions to 1) the point-set matching problem without correspondence and 2) the free-form curve matching problem. The algorithm requires no extracted features, no curve or surface derivatives, and no preprocessing of 3-D data, except for the removal of statistical outliers.

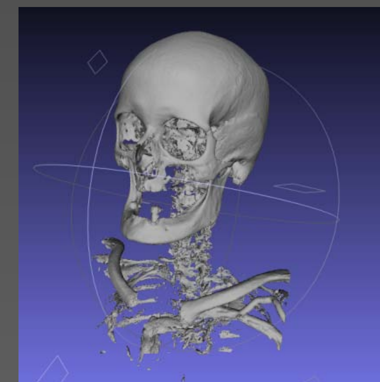
The main application of the proposed method as described

Fractional ICP (FICP)

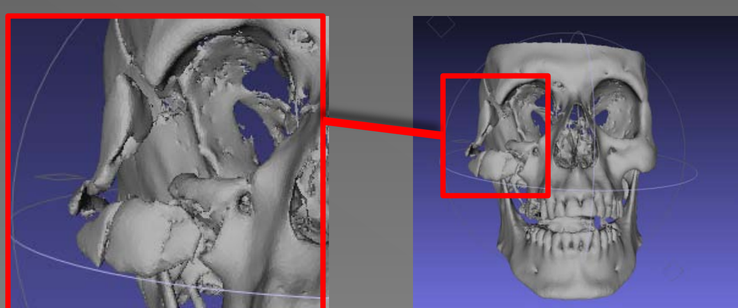
- Phillips J. M., Liu R., and Tomasi C. Outlier robust ICP for minimizing fractional RMSD. In *Proc. of Int. Conf. on 3D Digital Imaging and Modeling* (2007), pp. 427–434.

Medical Data

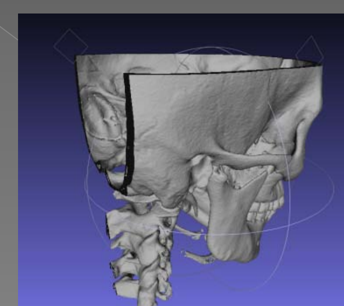




- complete and abnormal



Fracture

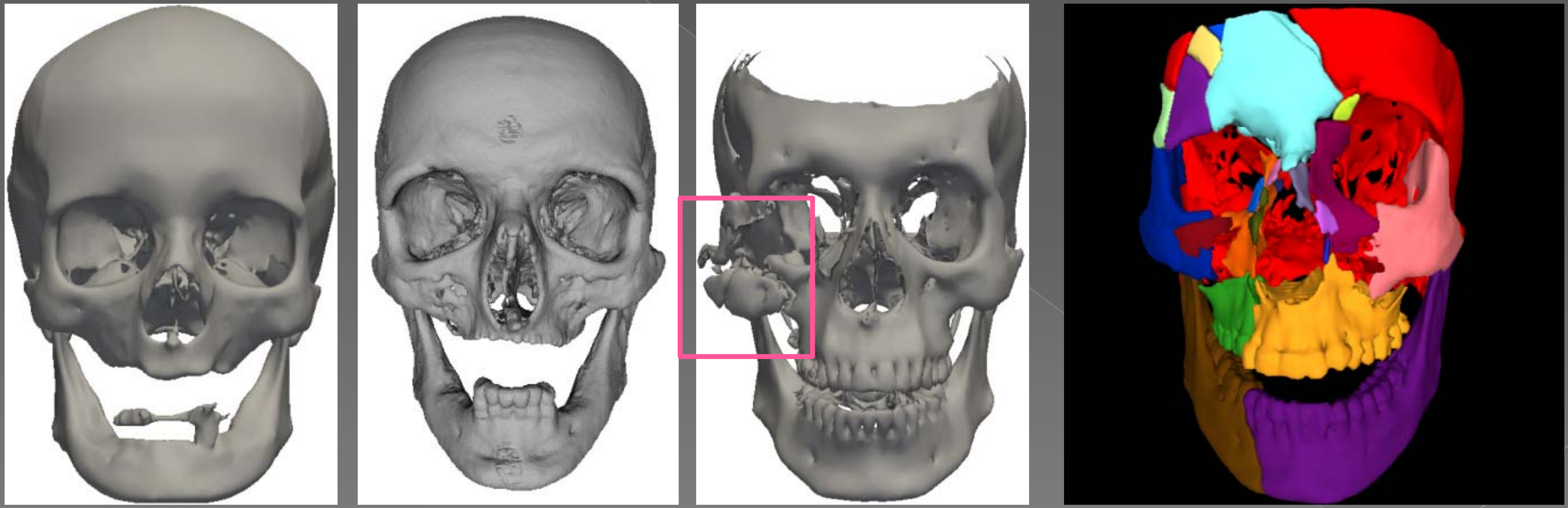


Incomplete scanning

Medical Data

- Models to be registered:

  vary greatly in size, shape details and completeness.



Idea

- Identify the outliers
  - Reject wrong pairs by distance between that paired points.

Fractional root mean distance

- Problem formulation:
  - p: a point on the object
  - D: object point set
  - D_f: is a subset of D (inliers)
  - f: $\frac{|D_f|}{|D|}$, [0,1]
  - M: model point set
  - μ: closest point function

$$\mathrm{FRMSD}(D, M, f, \mu) = \frac{1}{f^{\lambda}} \sqrt{\frac{1}{|D_f|} \sum_{p \in D_f} ||p - \mu(p)||^2}$$

FICP

$$\mathrm{RMSD}(D, M, \mu) = \sqrt{\frac{1}{|D|}\sum_{p \in D} ||p - \mu(p)||^2}$$

- Repeat until convergence:
  - Compute $\mu: D \to M$ that minimizes RMSD
  - Compute f and D_f that minimizes
    - fractional root mean squared distance

$$\mathrm{FRMSD}(D, M, f, \mu) = \frac{1}{f^{\lambda}}\sqrt{\frac{1}{|D_f|}\sum_{p \in D_f} ||p - \mu(p)||^2}$$

  - Compute transformation T that minimizes RMSD on D_f

Choose a point set with less RMSD

Choose a point set with more points

Find f and D_f

- Compute the distance $r = ||p - \mu(p)||$.
- Sort the points in increasing order of this r.
- Computer the error for the $|D|$ possible values of f.
- Choose the f that resulted in the smallest error.

$$O(|D|)$$

Converge to local minimum

smaller FRMSD

- Repeat until convergence:
  - Compute μ: D -> M that minimizes RMSD
  - Compute f and D_f that minimizes
    - fractional root mean squared distance

smaller FRMSD

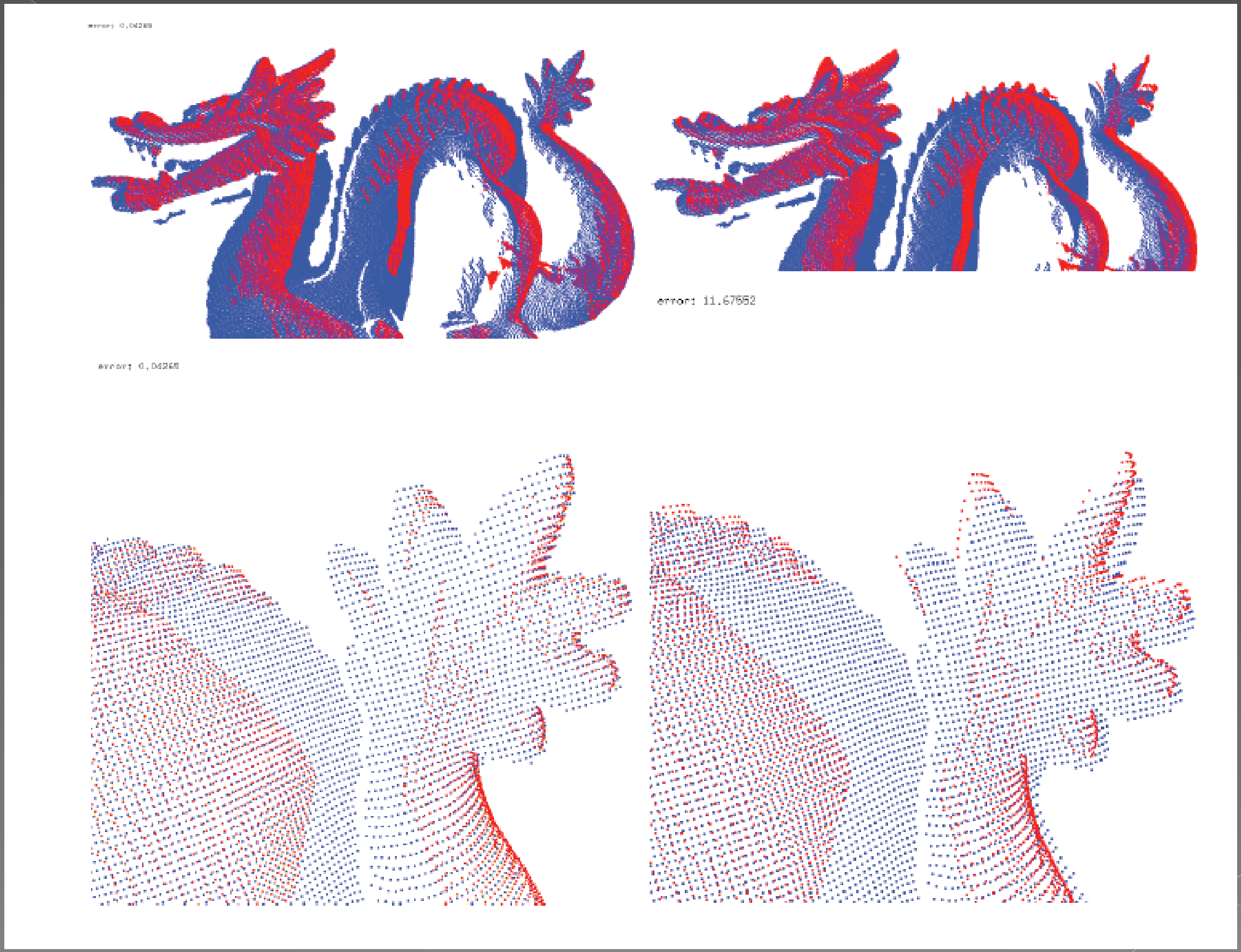
$$\mathrm{FRMSD}(D, M, f, \mu) = \frac{1}{f^{\lambda}} \sqrt{\frac{1}{|D_f|} \sum_{p \in D_f} ||p - \mu(p)||^2}$$

  - Compute T that minimizes RMSD on D_f

smaller FRMSD

Results

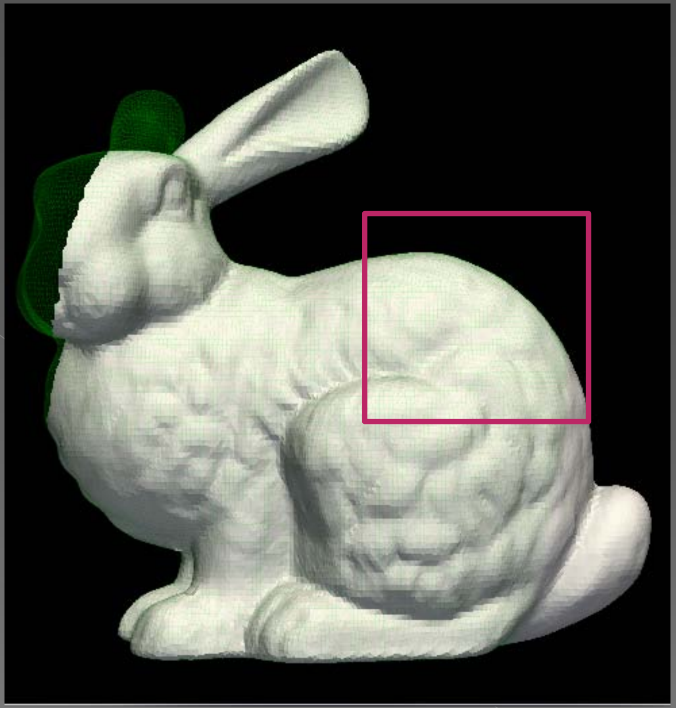
Dragon Model



FICP ICP

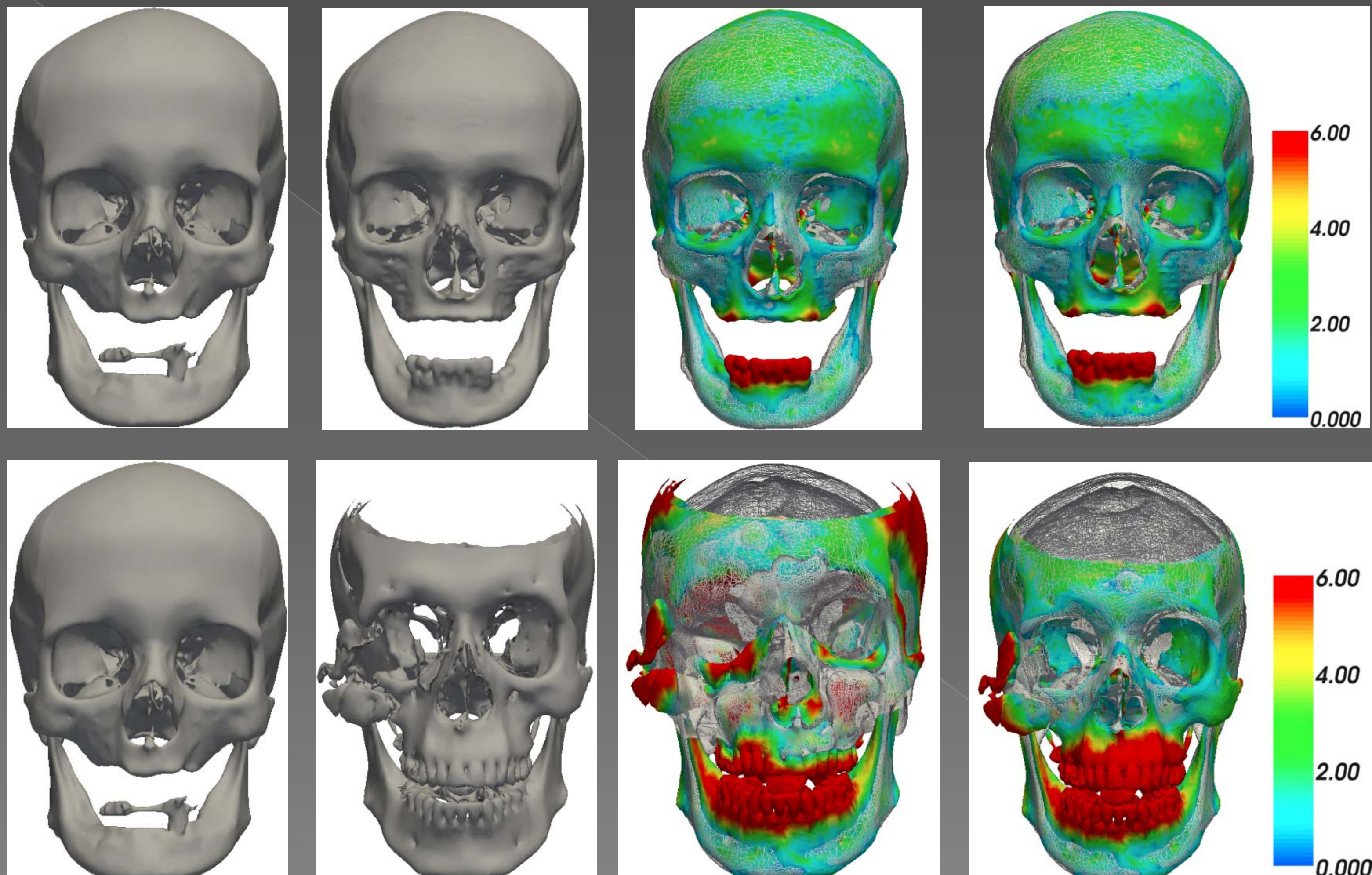
Bunny model

ICP



FICP

Skull models



Complete

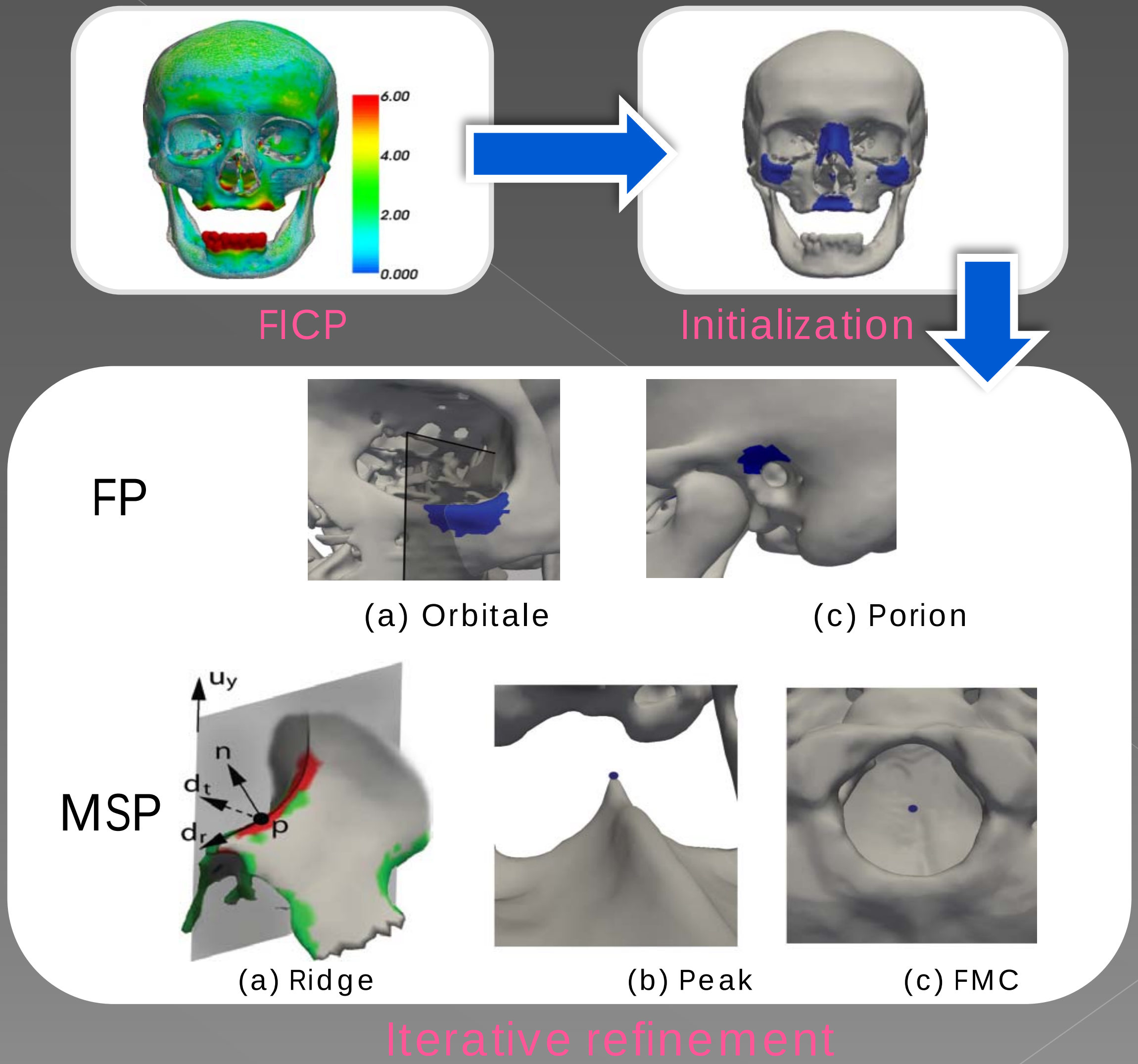
Patient

Source | Target | ICP result | FICP result

Application

Automatic Midline Landmark Identification

Cheng Yuan, Leow Wee Kheng, Lim Tiam Chye: Automatic Identification of Frankfurt Plane and Mid-Sagittal Plane of Skull. In WACV 2012, pp. 233-238.



Reference

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- J.M.Phillips, R. Liu, C. Tomasi. *Outlier Robust ICP for Minimizing Fractional RMSD.* International Conference on 3-D Digital Imaging and Modeling, 2007.
- Horn, B.K.P. Closed-form solution of absolute orientation using unit quaternions. The Journal of the Optical Society of America A, 4(4):239–256, 1987.
- Horn, B.K.P. Closed-form solution of absolute orientation using orthonormal matrices. The Journal of the Optical Society of America A, 5(7):1127--1135, 1988.