CS1010 Programming Methodology

http://www.comp.nus.edu.sg/~cs1010/

Unit 20

Searching and Sorting

NUS National University of Singapore | School of Computing

Unit 20: Searching and Sorting

Objectives:

- Understand the basic searching algorithms and sorting algorithms
- Introduce the concept of complexity analysis (informally)
- Implement the searching and sorting algorithms using arrays

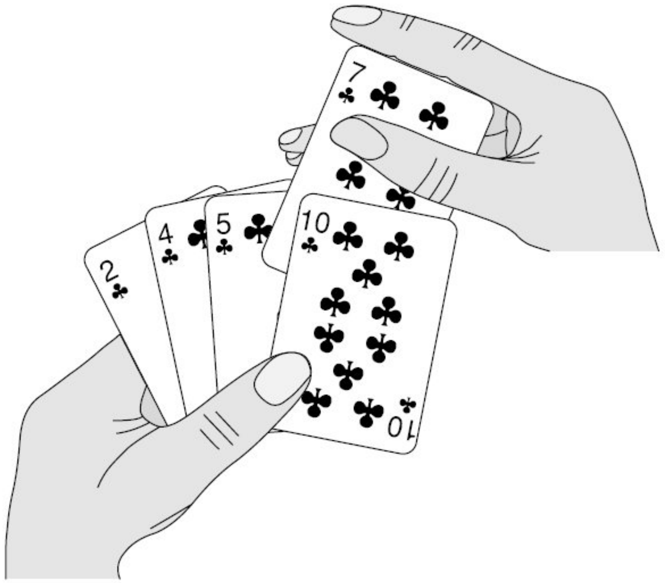
Reference:

- Chapter 7 Array Pointers
  - Section 7.6 Searching and Sorting an Array

Unit 20: Searching and Sorting (1/2)

1. Overall Introduction
2. Introduction to Searching
3. Linear Search
   - Demo #1
   - Performance
4. Binary Search

SEARCHING

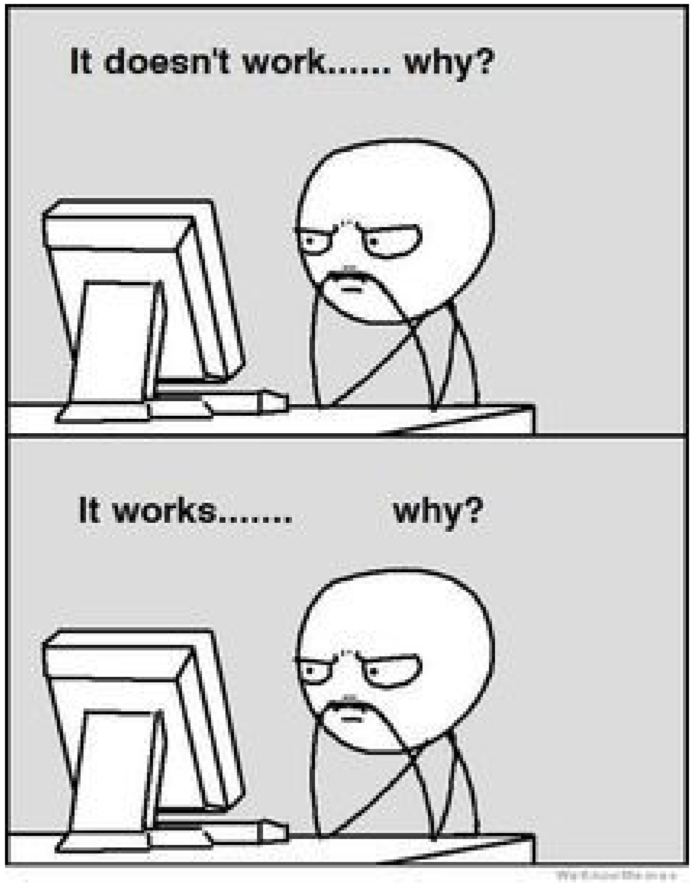


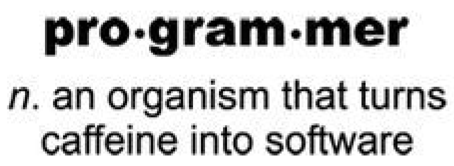
Unit 20: Searching and Sorting (2/2)

5. Introduction to Sorting
6. Selection Sort
   - Demo #2
   - Performance
7. Bubble Sort
   - Demo #3
   - Performance
8. More Sorting Algorithms
9. Animated Sorting Algorithms

1. Overall Introduction

- You have accumulated quite a bit of basic programming experience by now.




pro·gram·mer

n. an organism that turns caffeine into software

1. Overall Introduction

- You have accumulated quite a bit of basic programming experience by now.
- Today, we will study some simple yet useful classical algorithms which find their place in many CS applications
  - Searching for some data amid very large collection of data
  - Sorting very large collection of data according to some order
- We will begin with an algorithm (idea), and show how the algorithm is transformed into a C program (implementation).
- This brings back (reminds you) our very first lecture: the importance of beginning with an algorithm.

2. Introduction to Searching (1/2)

- Searching is a common task that we carry out without much thought everyday.
  - Searching for a location in a map
  - Searching for the contact of a particular person
  - Searching for a nice picture for your project report
  - Searching for a research paper required in your course
- In this lecture, you will learn how to search for an item (sometimes called a search key) in an array.

2. Introduction to Searching (2/2)

- Problem statement:

  Given a list (collection of data) and a search key X, return the position of X in the list if it exists.

  For simplicity, we shall assume there are no duplicate values in the list.

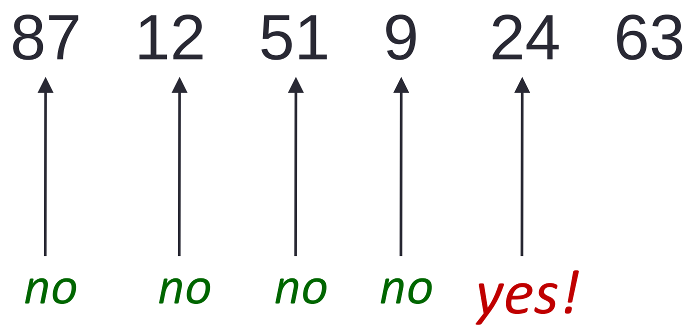
- We will count the number of comparisons the algorithms make to analyze their performance.
  - The ideal searching algorithm will make *the least possible number of comparisons* to locate the desired data.
  - We will introduce worst-case scenario.
  - (This topic is called analysis of algorithms, which will be formally introduced in CS2040. Here, we will give an informal introduction just for an appreciation.)

3. Linear Search (1/3)

- Also known as Sequential Search
- Idea: Search the list from one end to the other end in linear progression.
- Algorithm:

```
// Search for key in list A with n items
linear_search(A, n, key)
{
   for i = 0 to n-1
        if $A_i$ is key then report i
}
```

Example: Search for **24** in this list



Return 4

- Question: What to report if key is not found?
  - Aim for a clean design

3. Linear Search: Demo #1 (2/3)

- If the list is an array, how would you implement Linear Search algorithm?

```c
// To search for key in arr using linear search
// Return index if found; otherwise return -1
int linearSearch(int arr[], int size, int key) {
    int i;

    for (i=0; i<size; i++)
        if (key == arr[i])
            return i;
    return -1;
}
```

See linear_search.c for full program

Useful and common technique

- Question: What if array contains duplicate values of the key?

Index of the first element found will be returned.

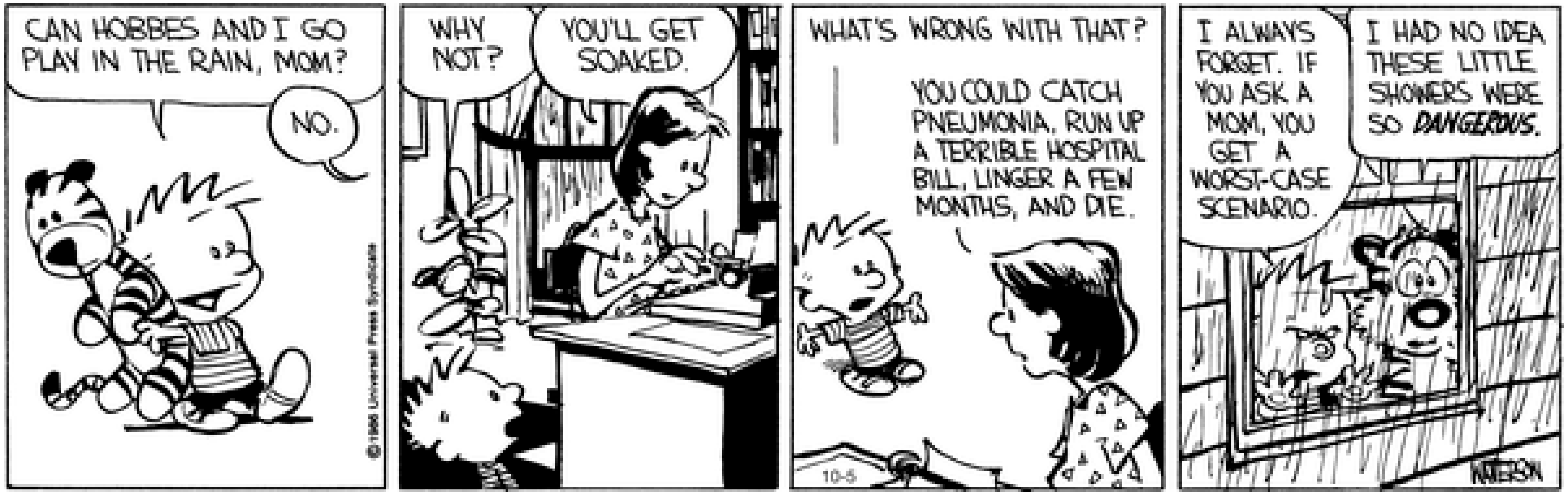
3. Linear Search: Performance (3/3)

- We use the number of comparisons here as a rough basis for measurement.
- Analysis is done for best case, average case, and worst case. We will focus on the worst case.
- For an array with *n* elements, in the worst case,
  - What is the number of comparisons in linear search algorithm?

    n comparisons
    This is called **linear time** algorithm: O(*n*)
  - Under what circumstances does the worst case happen?

    (a) Not found
    (b) Found at the last element

Worst-case Scenario


CAN HOBBES AND I GO PLAY IN THE RAIN, MOM?
NO.
WHY NOT?
YOU'LL GET SOAKED.
WHAT'S WRONG WITH THAT?
YOU COULD CATCH PNEUMONIA, RUN UP A TERRIBLE HOSPITAL BILL, LINGER A FEW MONTHS, AND DIE.
I ALWAYS FORGET. IF YOU ASK A MOM, YOU GET A WORST-CASE SCENARIO.
I HAD NO IDEA THESE LITTLE SHOWERS WERE SO DANGEROUS.
WATTERSON

4. Binary Search (1/6)

- You are going to witness a radically different approach, one that has become the basis of many well-known algorithms in Computer Science!
- The idea is simple and fantastic, but when applied on the searching problem, it has this pre-condition that the list must be sorted before-hand.
- How the data is organized (in this case, sorted) usually affects how we choose/design an algorithm to access them.
- In other words, sometimes (actually, very often) we seek out new way to organize the data so that we can process them more efficiency. → More of this in CS2040 Data Structures and Algorithms.

Being Organized…

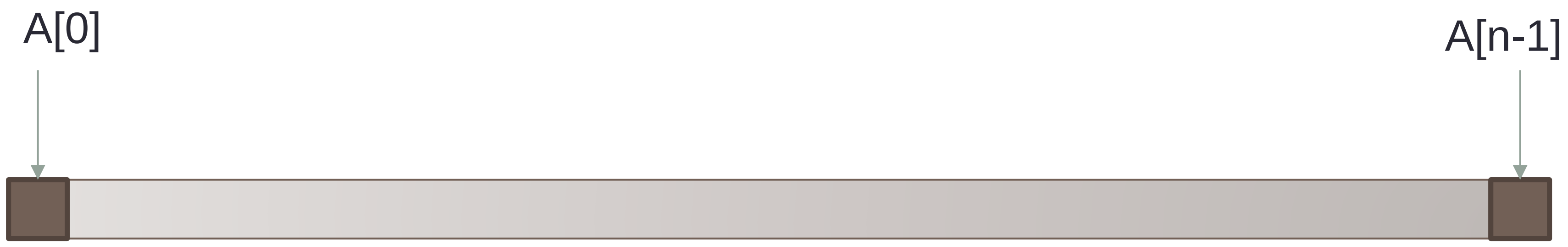
This is my room
What I think it looks like
What my mom thinks it looks like
What she describes it looks like
WeKnowMemes

4. Binary Search (2/6)

- The Binary Search algorithm
  - Look for the key in the middle position of the list. Either of the following 2 cases happens:
    - **If the key is smaller than the middle element, “discard” the right half of the list and repeat the process.**
    - **If the key is greater than the middle element, “discard” the left half of the list and repeat the process.**
  - Terminating condition: when the key is found, or when all elements have been “discarded”.

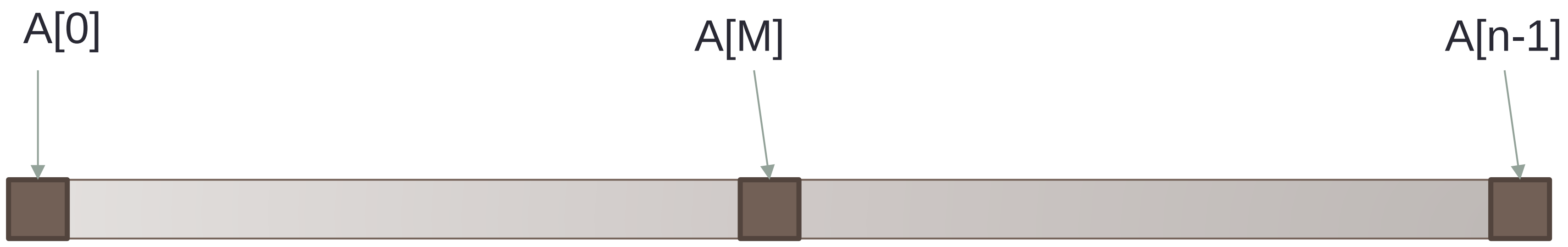
Binary Search

- Given a sorted array: A with n elements
- Goal: Search for x
  - x may or may not be in the array A



Binary Search

- Goal: Search for x
- Check the middle one
  - With index M = (0 + n-1) / 2



Binary Search

- Goal: Search for x
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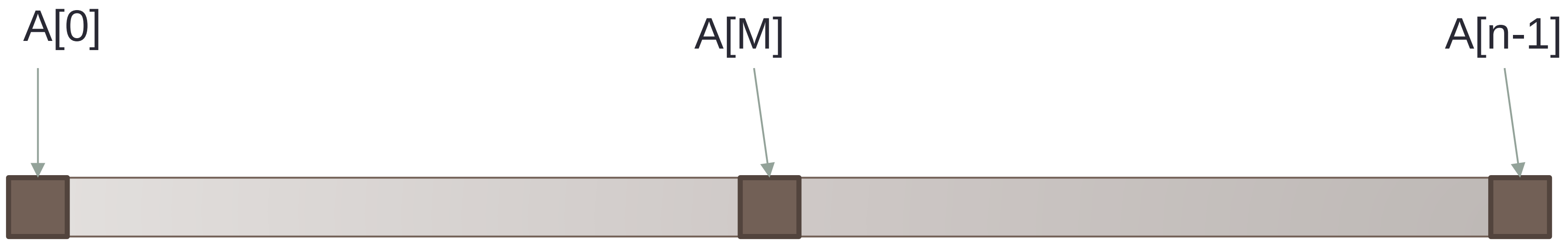
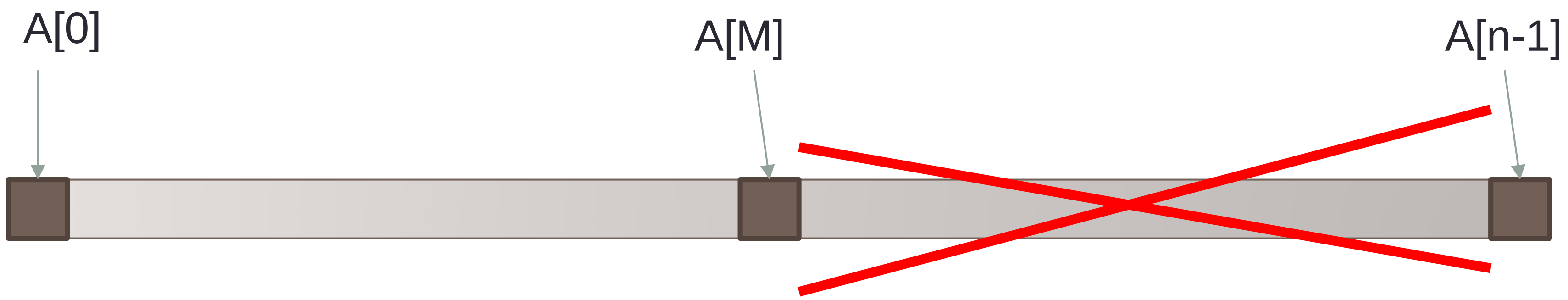


- If A[M] == x
  - Yeah.. we are done. x is found!

Binary Search

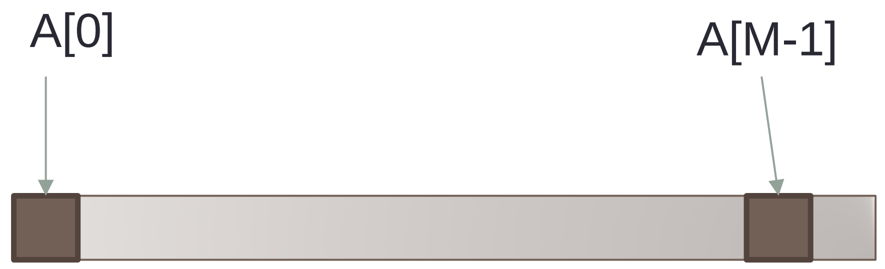
- Goal: Search for x
- Check the middle one
  - With index M = (0 + n-1) / 2



- If A[M] > x
  - Then all of the right side with indice M+1 to n-1 will be all larger than x
  - No need to search

Binary Search

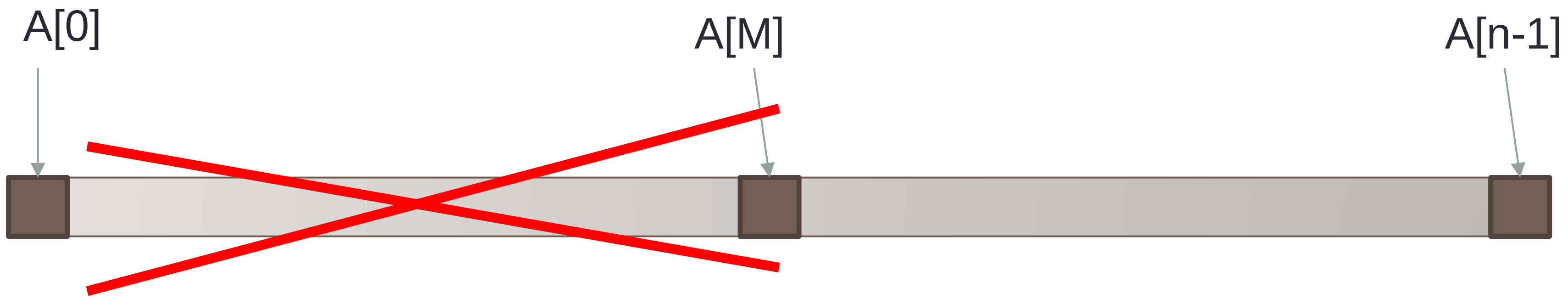
- Goal: Search for x
- Check the middle one
  - With index M = (0 + n-1) / 2



- If A[M] > x
  - Repeat the process with this smaller array with indices from 0 to M-1

Binary Search

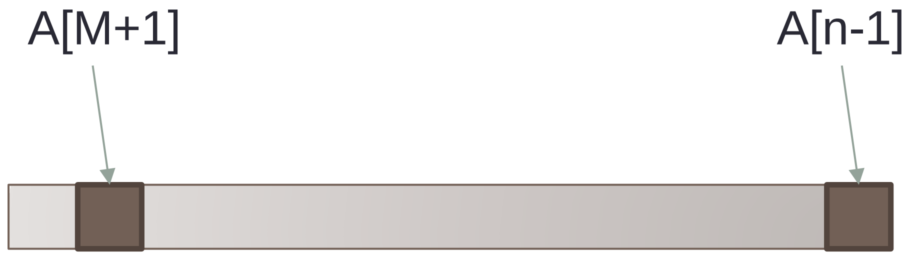
- Goal: Search for x
- Check the middle one
  - With index M = (0 + n-1) / 2



- If A[M] < x
  - Then all of the left side with indices 0 to M-1 will be all smaller than x
  - No need to search

Binary Search

- Goal: Search for x
- Check the middle one
  - With index M = (0 + n-1) / 2



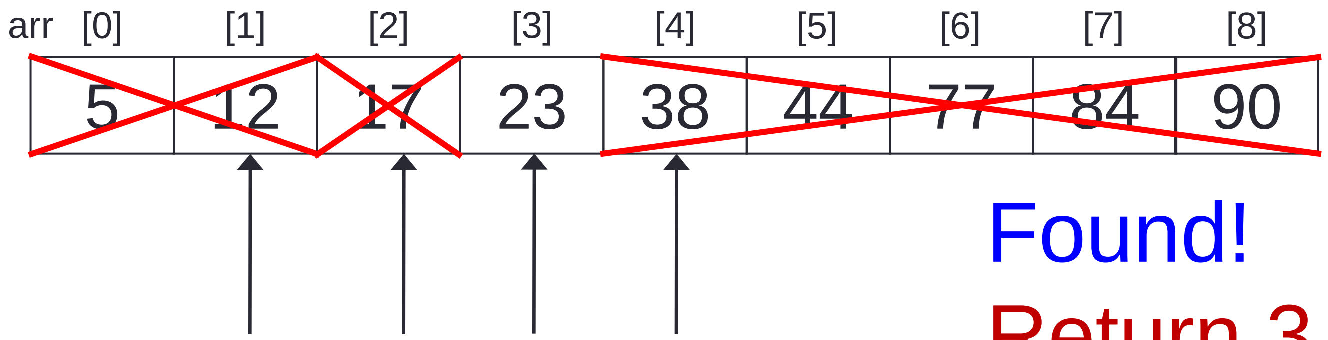
- If A[M] < x
  - Repeat the process with this smaller array with indices from M+1 to n-1

Pseudocode for Array Size = N

- Let it be L = 0, R = N-1
  - L and R are the two indices of the left and right bound
- Repeat
  - The middle index M = (L + R) / 2
  - If A[M] == x
    - Return the index M
  - If A[M] > x
    - R = M -1
      - Else
    - L = M +1
- Repeat until the gap is closed
  - x is not found in the array

4. Binary Search (3/6)

- Example: Search key = 23



Found!
Return 3

1. low = 0, high = 8, mid = (0+8)/2 = 4 arr[4] = 38 > 23
2. low = 0, high = 3, mid = (0+3)/2 = 1 arr[1] = 12 < 23
3. low = 2, high = 3, mid = (2+3)/2 = 2 arr[2] = 17 < 23
4. low = 3, high = 3, mid = (3+3)/2 = 3 arr[3] = 23 == 23

4. Binary Search (4/6)

- In binary search, each step eliminates the problem size (array size) by half!
  - The problem size gets reduced to 1 very quickly! (see slide after next.)
- This is a simple yet powerful strategy, of halving the solution space in each step
  - This is a BIG DEAL in problem solving (remember the Santa Claus' dirty socks problem in your first discussion session?)
- Such strategy, a special case of divide-and-conquer paradigm, can be naturally implemented using recursion.
- But for now, we will stick to an iterative solution (loop). (We will see its implementation using recursion when we cover recursion.)

4. Binary Search (5/6)

binary_search.c

```
// To search for key in sorted arr using binary search
// Return index if found; otherwise return -1
int binarySearch(int arr[], int size, int key) {

    int low = 0, high = size – 1, mid = (low + high)/2;

    while ((low <= high) && (arr[mid] != key)) {
        if (key < arr[mid])
            high = mid - 1;
        else
            low = mid + 1;
        mid = (low + high)/2;
    }
    if (low > high) return -1;
    else            return mid;

}
```

4. Binary Search (6/6)

- Worst-case analysis

Array size *n*	Linear Search (*n* comparisons)	Binary search ($\log_2 n$ comparisons)
100	100	≈7
1,000	1,000	≈10
10,000	10,000	≈14
100,000	100,000	≈17
1,000,000	1,000,000	≈20
10^9	10^9	≈30

This is called **$\log_2 n$** algorithm, or $O(\log_2 n)$

Why log N?

- If you have N items, and each time you reduce the search space by half
- How many steps do you need to half until the search space is only 1?
  - Let S be the number of steps
    - $N \times (\frac{1}{2})^S = 1$
    - $N = 2^S$
    - $\log N = S \log 2$
    - $S = \log N / \log 2$

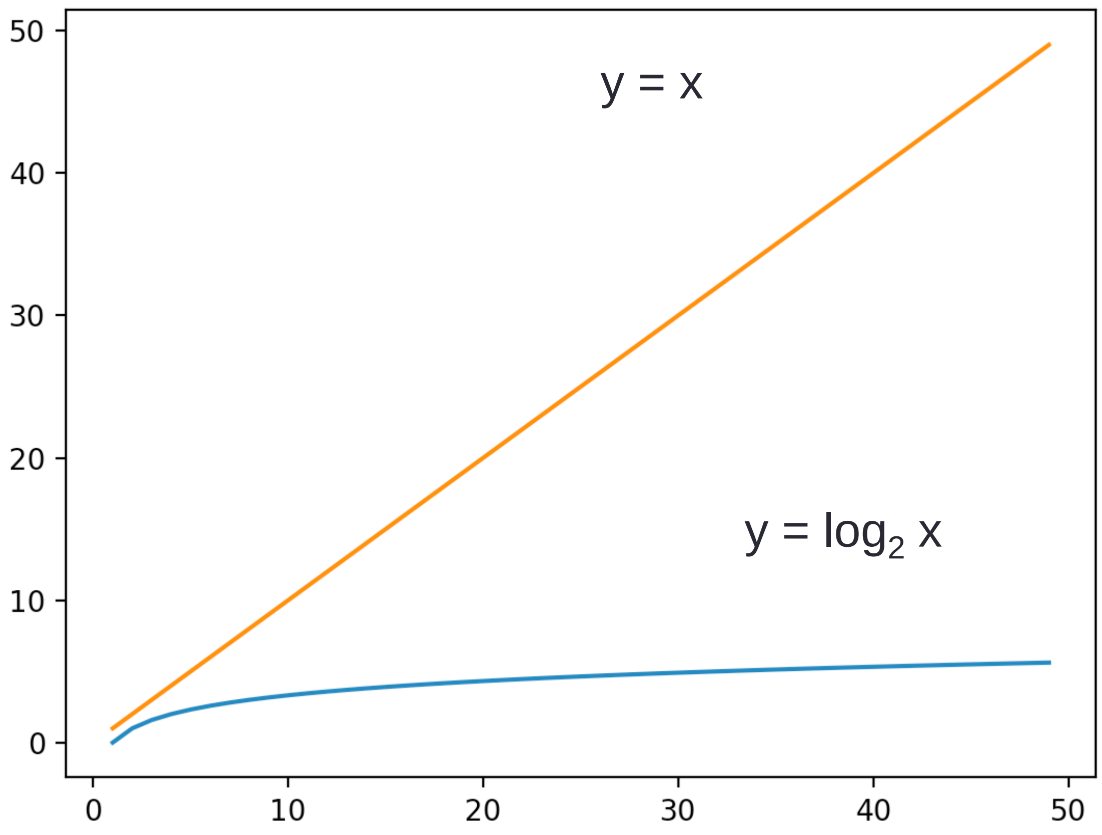
Order of Growth

Linear

Worst case: N

Binary Search

Log_2 N



Ok, now I know how to search a number quickly

Computation

- Usually we rely on the computer to do tasks that are
  - repetitive
    - e.g. computing a function with series like cosine

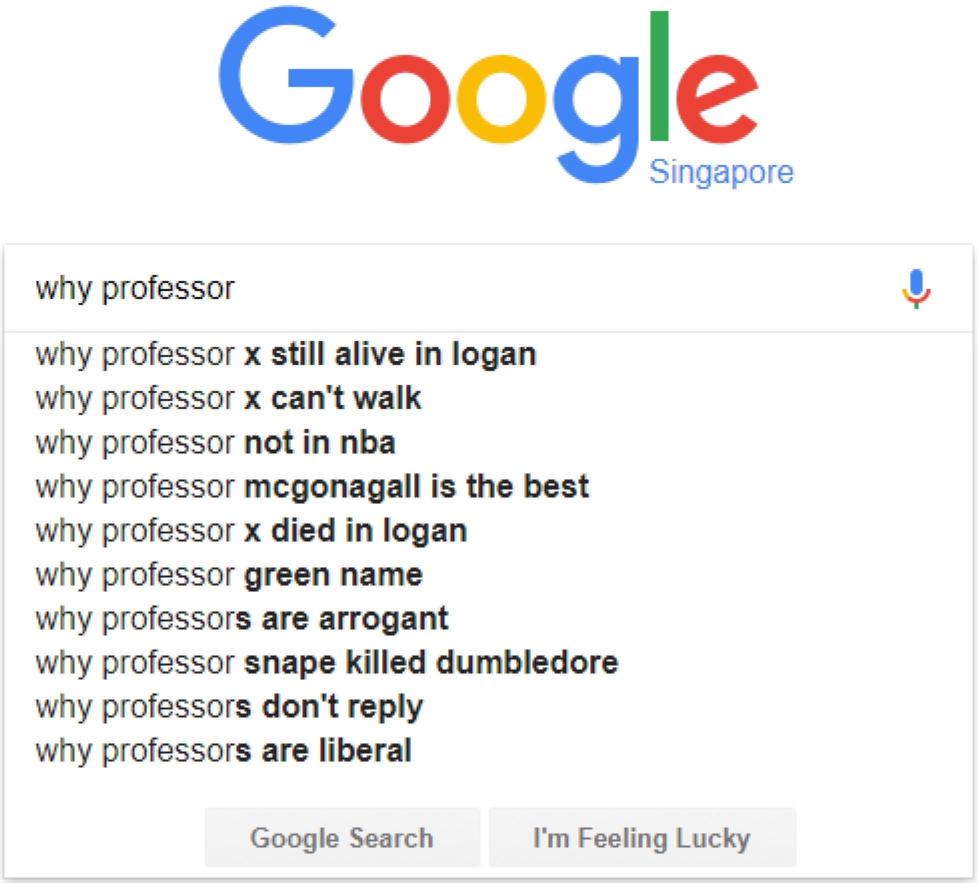
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

  - involving a large volume of data
    - Searching for a NRIC in the many millions of Singaporeans

Search?

- Simple problem
- Let’s say we have a file containing all the names of Singaporeans
  - 3.4millions in 2015
- You want to check
  - Is “Alan Cheng Ho Lun” a Singaporean?

Google?



why professor

why professor **x still alive in logan**
why professor **x can't walk**
why professor **not in nba**
why professor **mcgonagall is the best**
why professor **x died in logan**
why professor **green name**
why professor**s are arrogant**
why professor **snape killed dumbledore**
why professor**s don't reply**
why professor**s are liberal**

How much data does Google handle?

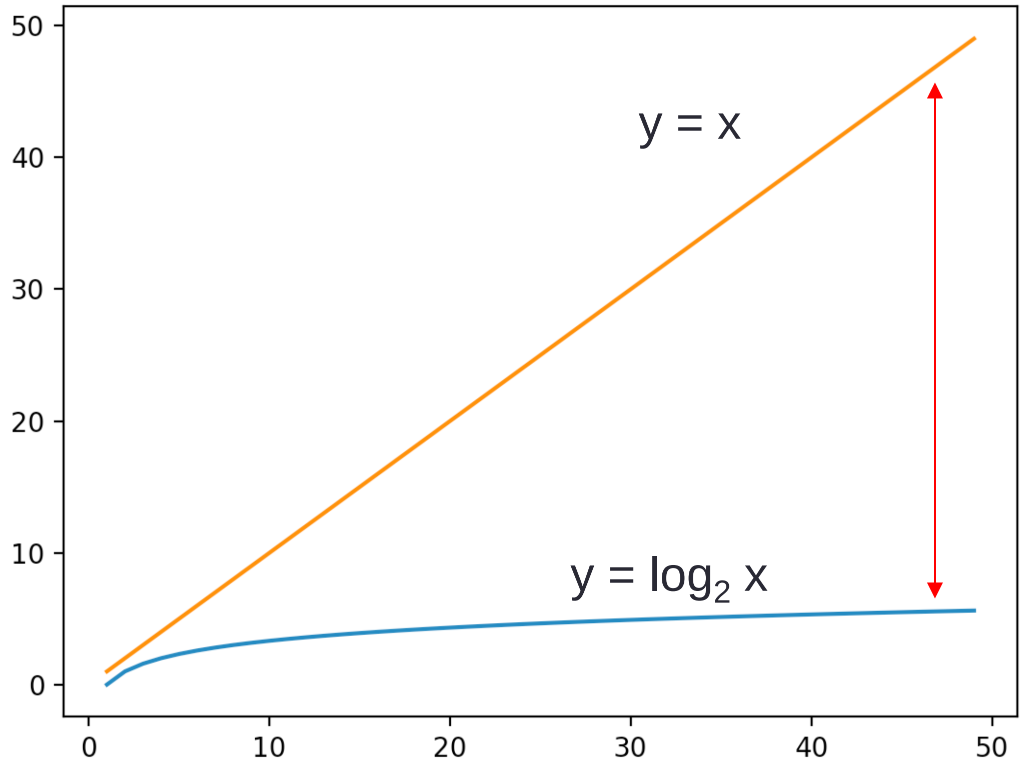
- About 10 to 15 Exabyte of data
  - 1 Exabyte(EB)= 1024 Petabyte(PB)
  - 1 Petabyte(PB) = 1024 Terabytes(TB)
  - 1 Terabyte(PB) = 1024 Gigabytes(TB)
    - = 4 X 256GB iPhone
- So Google is handling about 60 millions of iPhones

Let’s calculate

- 400Mb of data needs 6 seconds
  - I did an “anyhow” program to generate a lot of numbers worth 400Mb of data
- 15 Exabyte of data needs how long?
  - 15 EB = 15 x 1024 x 1024 x 1024 x 1024 MB
  - To search through 15EB of data…..
    - 7845 years…..
- If we do it with Binary Search
  - $\log_2$ (15EB) = 43 steps!!!!!

Speed Improvement

- If the number of data N is even larger, the improvement in speed is even greater



The Magic?

- Binary search vs Linear search
- However, one step back
  - The data has to be sorted
  - The time needed for sorting is $n \log n$
  - Slower than linear search
  - But we only have to do it once
  - Is it?
    - Can we assume that we can sort it once for all?

Data Structure and Algorithm

- But what if the data is dynamic?
- Even after sorted, there will be new items added or old items removed from the list?
- How do we maintain a dynamic list that is sorted all the time?
- This is the study of Data Structure
  - Basically, how fast we can compute if the data size N is very large

Data Structure

- How do you organize your data to improve your performance
  - Sorted list, trees, graphs, heaps, hash tables, etc.

- British Library
  - 170m+
- Library of Congress
  - 164m+

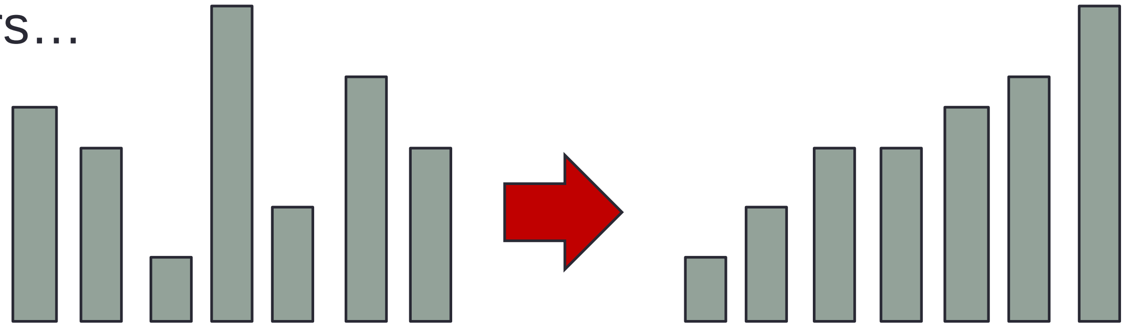
Algorithm

- What is the way I do my computation?
- Named for al-Khwarizmi (780-850)
    - Persian mathematician

Sorting

5. Introduction to Sorting (1/2)

- Sorting is any process of arranging items in some sequence and/or in different sets – Wikipedia.
- Sorting is important because once a set of items is sorted, many problems (such as searching) become easy.
  - Searching can be speeded up. (From linear search to binary search)
  - Determining whether the items in a set are all unique.
  - Finding the median item in a list.
  - Many others…



5. Introduction to Sorting (2/2)

- Problem statement:

  Given a list of *n* items, arrange the items into ascending order.

- We will implement the list as an integer array.
- We will introduce two basic sort algorithms.
- We will count the number of comparisons the algorithms make to analyze their performance.
  - The ideal sorting algorithm will make the least possible number of comparisons to arrange data in a designated order.
- We will compare the algorithms by analyzing their worst-case performance.

6. Selection Sort (1/6)

- Selection Sort algorithm

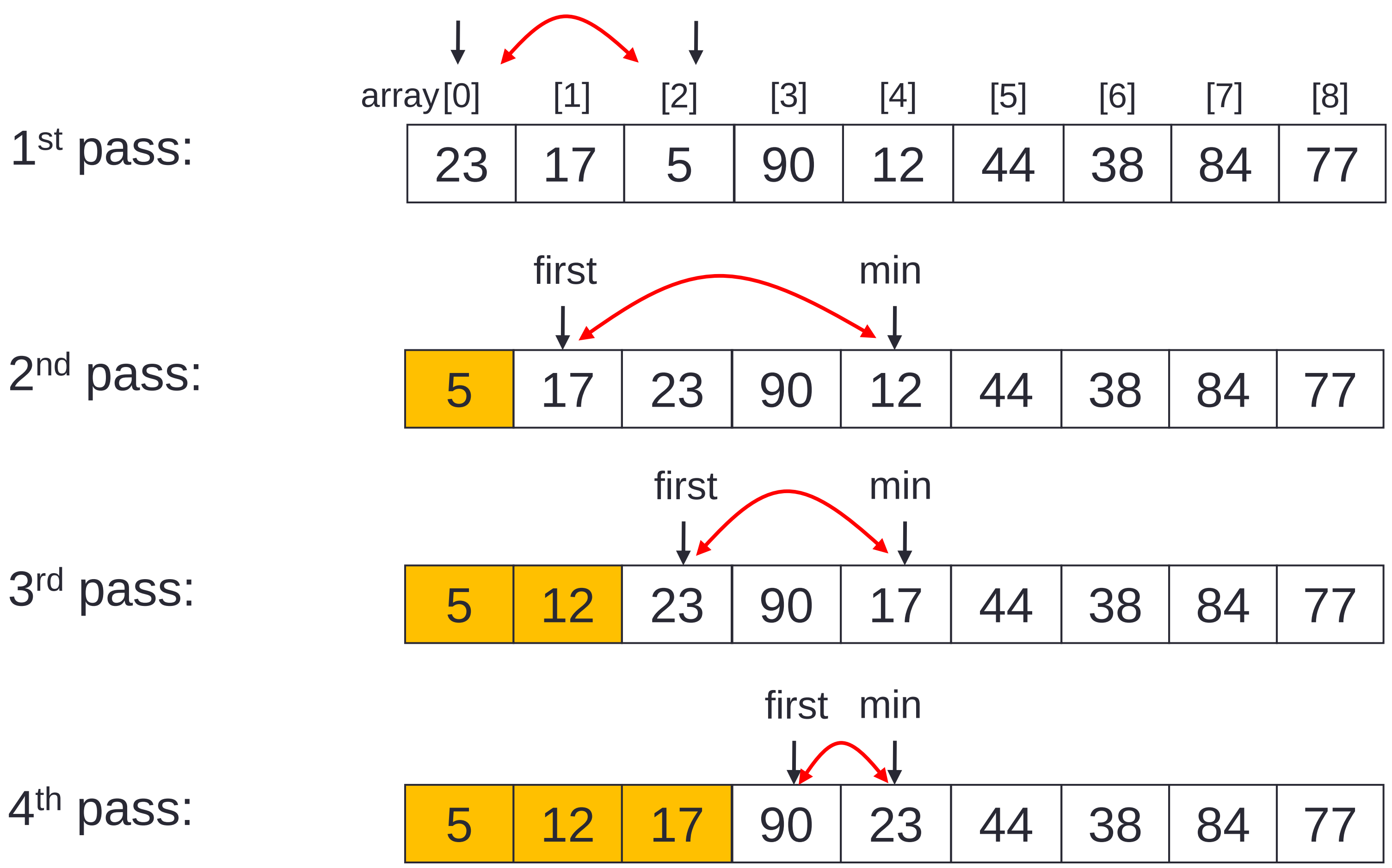
  Step 1: Find the smallest element in the list (find_min)

  Step 2: Swap this smallest element with the element in the first position. (Now, the smallest element is in the right place.)

  Step 3: Repeat steps 1 and 2 with the list having one fewer element (i.e. the smallest element just found and its place is “discarded” from further processing).

6. Selection Sort (2/6)

$n = 9$



6. Selection Sort (3/6)

Q: How many passes for an array with *n* elements?

$n = 9$

n – 1 passes

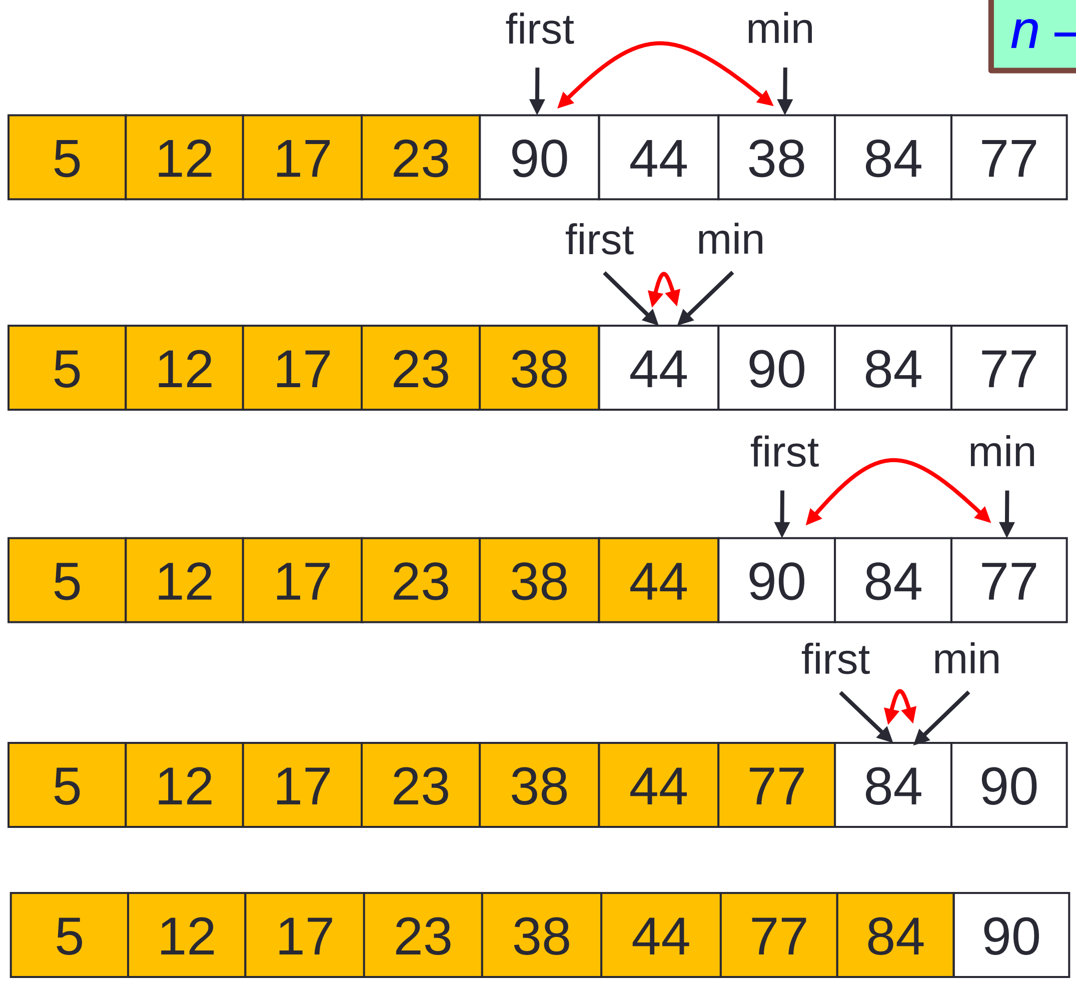
5th pass:

6th pass:

7th pass:

8th pass:

Final array:



6. Selection Sort: Demo #2 (4/6)

See selection_sort.c for full program

```c
// To sort arr in increasing order
void selectionSort(int arr[], int size) {
    int i, start, min_index, temp;

    for (start = 0; start < size-1; start++) {
       // each iteration of the for loop is one pass

       // find the index of minimum element
       min_index = start;
       for (i = start+1; i < size; i++)
           if (arr[i] < arr[min_index])
               min_index = i;

       // swap minimum element with element at start index
       temp = arr[start];
       arr[start] = arr[min_index];
       arr[min_index] = temp;
    }
}
```

6. Selection Sort: Performance (5/6)

- We choose the number of comparisons as our basis of analysis.
- Comparisons of array elements occur in the inner loop, where the minimum element is determined.
- Assuming an array with *n* elements. Table below shows the number of comparisons for each pass.
- The total number of comparisons is calculated in the formula below.
- Such an algorithm is called an **O(n^2)** algorithm, or quadratic algorithm, in terms of running time complexity.

Pass	#comparisons
1	$n-1$
2	$n-2$
3	$n-3$
…	…
$n-1$	1

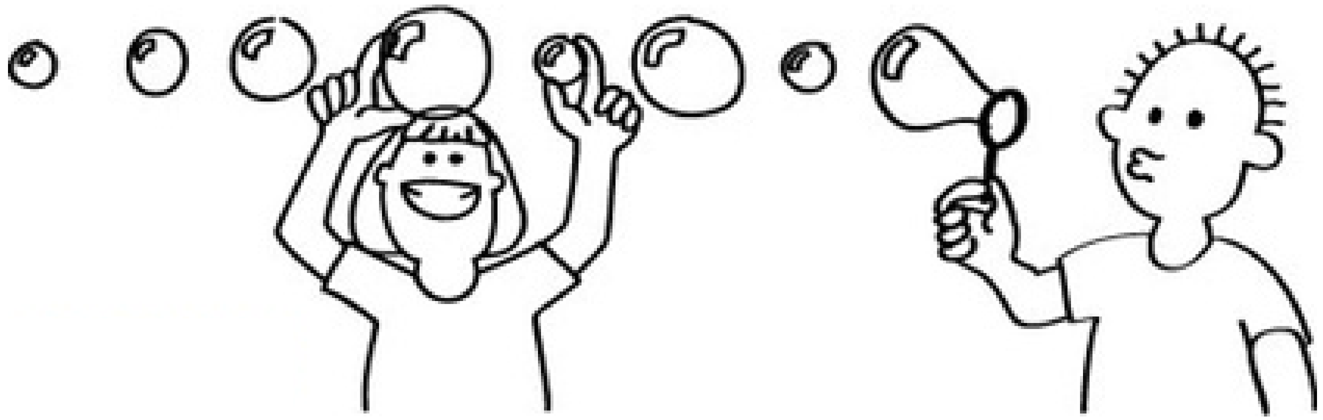
$$\sum_{i=1}^{n-1} i = \frac{(n-1)(n)}{2} = \frac{n^2-n}{2} \cong n^2$$

6. Selection Sort (6/6)

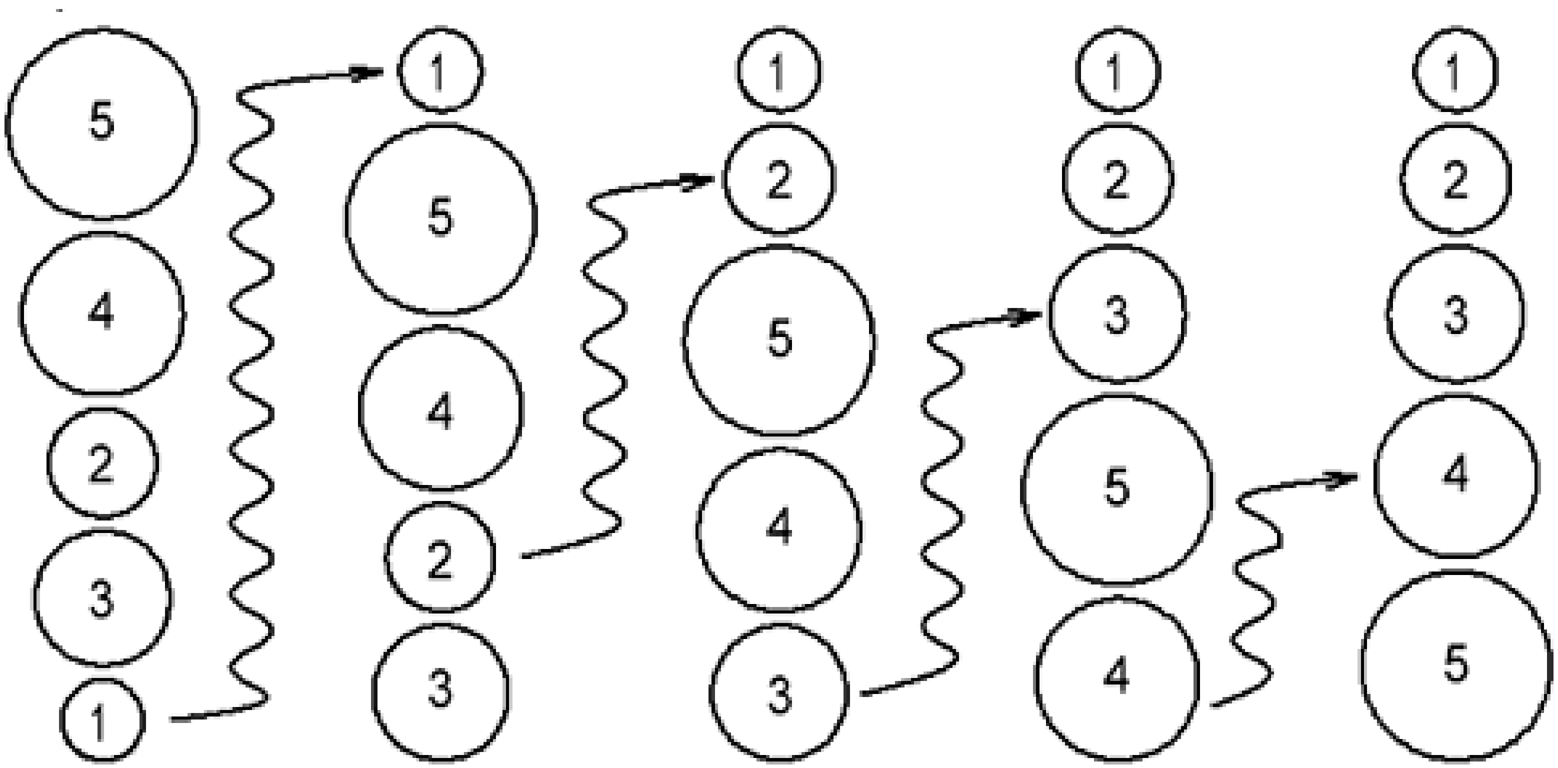
- Selection sort is classified under exchange sort, where elements are exchanged in the process.
- We could search for the minimum element as described earlier, or search for the maximum element and exchange it with the last element of the working array (assuming we sort in ascending order).

7. Bubble Sort (1/5)

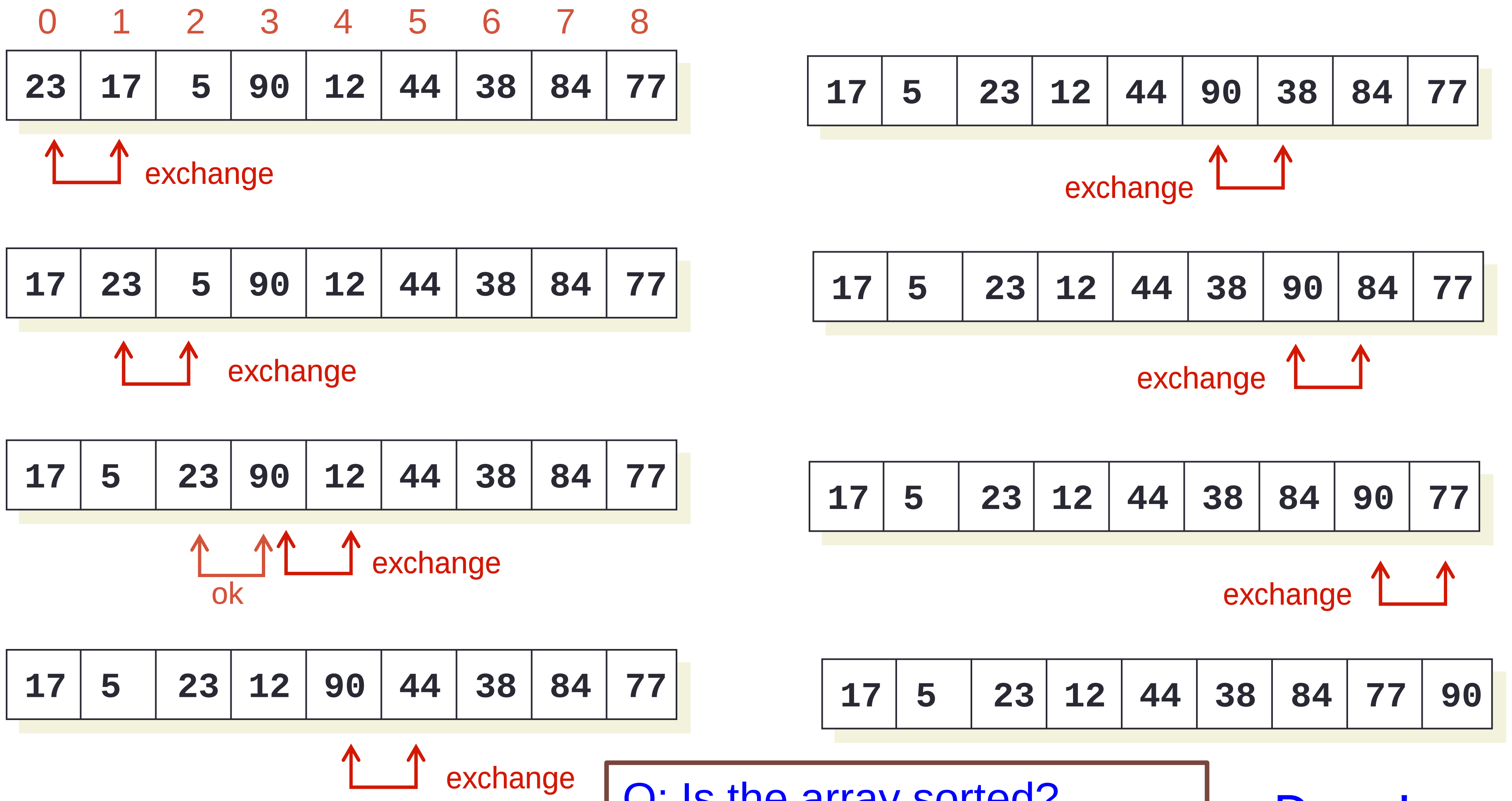
- Selection sort makes one exchange at the end of each pass.
- What if we make more than one exchange during each pass?
- The key idea Bubble sort is to make pairwise comparisons and exchange the positions of the pair if they are in the wrong order.



Idea: Like Bubbles



7. Bubble Sort: One Pass of Bubble Sort (2/5)



Q: Is the array sorted?
Q: What have we achieved?

Done!

7. Bubble Sort: Demo #3 (3/5)

See bubble_sort.c for full program

```
// To sort arr in increasing order
void bubbleSort(int arr[], int size) {
    int i, limit, temp;

    for (limit = size-2; limit >= 0; limit--) {
        // limit is where the inner loop variable i should end

        for (i=0; i<=limit; i++) {
            if (arr[i] > arr[i+1]) { // swap arr[i] with arr[i+1]
                temp = arr[i];
                arr[i] = arr[i+1];
                arr[i+1] = temp;
            }
        }
    }
}
```

7. Bubble Sort: Performance (4/5)

- Bubble sort, like selection sort, requires $n - 1$ passes for an array with n elements.
- The comparisons occur in the inner loop. The number of comparisons in each pass is given in the table below.
- The total number of comparisons is calculated in the formula below.
- Like Selection sort, Bubble sort is also an **O(n^2)** algorithm, or quadratic algorithm, in terms of running time complexity.

Pass	#comparisons
1	$n - 1$
2	$n - 2$
3	$n - 3$
…	…
$n - 1$	1

$$\sum_{i=1}^{n-1} i = \frac{(n-1)(n)}{2} = \frac{n^2 - n}{2} \cong n^2$$

7. Bubble Sort: Enhanced version (5/5)

- It is possible to enhance Bubble sort algorithm to reduce the number of passes.
- Suppose that in a certain pass, no swap is needed. This implies that the array is already sorted, and hence the algorithm may terminate without going on to the next pass.
- You will implement this enhanced version in your discussion session.

8. More Sorting Algorithms

- We have introduced 2 basic sort algorithms: Selection Sort and Bubble Sort. Together with Insertion Sort algorithm, these 3 are the simplest sorting algorithms.
- However, they are very slow, as their running time complexity is quadratic, or O(n^2), where n is the array size.
- Faster sorting algorithms exist and are covered in more advanced modules.
- In CS2040, you will learn more advanced sorting algorithms with better running-time efficiency: Quick Sort, Merge Sort, Radix Sort, etc.

9. Animated Sorting Algorithms

- There are a number of animated sorting algorithms on the Internet.
- Here are just a few sites:
  - Visualgo (http://visualgo.net): Project under Dr Steven Halim
  - http://www.sorting-algorithms.com/
  - http://www.cs.ubc.ca/~harrison/Java/sorting-demo.html
- There are also folk dances based on sorting!
  - Selection sort with Gypsy folk dance
    http://www.youtube.com/watch?v=Ns4TPTC8whw
  - Bubble sort with Hungarian folk dance
    http://www.youtube.com/watch?v=lyZQPjUT5B4

k10555323 fotosearch.com

Summary

- In this unit, you have learned about
  - 2 search algorithms: Linear (sequential) Search and Binary Search
  - 2 basic sort algorithms: Selection Sort and Bubble Sort.