CS1231 Assignment #1
AY2018/19 Semester 1
Deadline: 5pm on Thursday, 20 September 2018

Instructions:

This is a **graded** assignment worth 10% of your final grade. Please work on it **by yourself**, not in a group or in collaboration with anybody.

Answer ALL questions. A handwritten submission is fine; there is no need to use Word or Latex to typeset. However, please write legibly, and use standard A4 foolscap papers. Also, write **your name, student number and tutorial group** at the top of the first page. To keep your submission short, you need not write/include the questions in your submittion. Staple all pages. Missing pages will cause you to lose marks.

Please note submission deadline above. Late submissions will not be graded.

How to submit:

Drop your submission into a slot labeled "CS1231 Assignment" at the Undergraduate Studies Office at COM1-02-19.

Question 1. (3 marks)

Translate the following English sentences into predicate logic statements. Note that marks may be deducted if your answer is not concise even though it might be correct.

You may leave out the domain of discourse in your statements (i.e. you do not need to specify what domain a variable belongs to, such as $\forall x \in D$; just write $\forall x$).

Use only the following predicates and no others:

- $Loves(x, y)$: x loves y
- $Reindeer(x)$: x is a reindeer

For example, "Aiken loves Dueet": $Loves(Aiken, Dueet)$.

(a) (1 mark) Everyone who loves Santa loves any reindeer.

(b) (1 mark) John loves Mary, and nobody else loves Mary.

(c) (1 mark) Anyone who loves reindeers loves at most one reindeer. (For example, if John loves Dasher and Mary loves Blitzen, then they wouldn't love any other reindeer. Do not use $\exists!$ for this question.)

Question 2. (3 marks)

Given below are three statements followed by three conclusions. Take the three statements to be true even if they deviate from commonly known facts.

Statements:

1. All actors are musicians.
2. No musician is a singer.
3. Some singers are dancers.

Conclusions:

1. Some actors are singers.
2. Some dancers are actors.
3. No actor is a singer.

Use only the following predicates: $Actor(x)$, $Musician(x)$, $Singer(x)$ and $Dancer(x)$.

(a) (1 mark) Write the quantified statements for statements (2) and (3). Statement (1) has been done for you: $\forall x(Actor(x) \rightarrow Musician(x))$.

(b) (2 marks) Choose among the three conclusions one that is valid, and show why it is so.

Question 3. (4 marks)

You are on the island of Wantuutrewan and you find two chests A and B in a cave.

On chest A is written: "At least one of these two chests contains a treasure." ... (W1)

On chest B is written: "Chest A contains a cobra." ... (W2)

You meet a knight on the island who gives you the following clues:

- Each of the chests either contains a treasure or a cobra but not both ... (C1)
- Either both the writings are true, or they are both false ... (C2)

Which chest should you open so that you are guaranteed to find a treasure in it?

Write your solution using propositional logic, justifying each step. You are given the following two propositions:

- a means "chest A contains a treasure."
- b means "chest B contains a treasure."

Question 4. (5 marks)

Recall the decimal representation of a nonnegative integer from Q6 of Tutorial 3:

> **Definition:** Given any nonnegative integer n, the **decimal representation** of n is an expression of the form
>
> $$d_k d_{k-1} \cdots d_2 d_1 d_0,$$
>
> where k is a nonnegative integer; $d_0, d_1, d_2, \ldots, d_k$ (called the **decimal digits** of n) are integers from 0 to 9 inclusive; $d_k \neq 0$ unless $n = 0$ and $k = 0$; and
>
> $$n = d_k \cdot 10^k + d_{k-1} \cdot 10^{k-1} + \cdots + d_2 \cdot 10^2 + d_1 \cdot 10 + d_0.$$
>
> (For example, $2{,}503 = 2 \cdot 10^3 + 5 \cdot 10^2 + 0 \cdot 10 + 3$.)

Define the sum of the digits of n by $SumDigits(n) = d_k + d_{k-1} + \cdots + d_1 + d_0$. Examples:

$SumDigits(5) = 5$.

$SumDigits(1234) = SumDigits(24013) = 10$.

$SumDigits(SumDigits(123456)) = 3$.

Some properties of $SumDigits$:

P1. $SumDigits(n) \equiv n \pmod 9$, for all $n \in \mathbb{N}$.

P2. $SumDigits(a+b) \leq SumDigits(a) + SumDigits(b)$, for all $a, b \in \mathbb{N}$.

P3. $SumDigits(ab) \leq SumDigits(a)SumDigits(b)$, for all $a, b \in \mathbb{N}$.

(a) (2 marks) Prove property P1.

(b) (3 marks) A nonnegative integer m is such that $SumDigits(m) = 100$, and $SumDigits(44m) = 800$. Without explicitly finding m, determine $SumDigits(3m)$.
(No marks will be given if you explicitly find m.)

Question 5. (5 marks)
$n > 0$ red balls and n blue balls are arranged to form a circle. You walk around the circle exactly once in a clockwise direction and count the number of red and blue balls you pass. If at all times during your walk, the number of red balls (that you have passed) is greater than or equal to the number of blue balls (that you have passed), then your trip is said to be *successful*. (Note that whether successful or not, you will pass exactly $2n$ balls after walking one round.)

Define predicate $P(n) =$ (In any circle formed by n red and n blue balls, there exists a successful trip), $\forall n \in \mathbb{Z}^+$.

Using the above predicate $P(n)$, prove by Mathematical Induction (either Regular or Strong Induction) that you can always make a successful trip if you can choose where you start. (Note: you must use $P(n)$ as the induction predicate in your proof. You may not modify it, nor use a different predicate.)

Reminder: Have you written your **name, student number and tutorial group** at the top of the first page? Your tutorial group is important as we will be passing your script to your tutor for grading. You need to write your tutorial group number in your mid-term test as well, so remember your tutorial group number well.

—— **END** ——