CS1231 Assignment #2
AY2018/19 Semester 1
Deadline: 5pm on Thursday, 1 November 2018

Instructions:

This is a **graded** assignment worth 10% of your final grade. Please work on it **by yourself**, not in a group or in collaboration with anybody.

Answer ALL questions. A handwritten submission is fine; there is no need to use Word or Latex to typeset. However, please write legibly, and use standard A4 foolscap papers. Also, write **your name, student number and tutorial group** at the top of the first page. To keep your submission short, you need not write/include the questions in your submission. Staple all pages. Missing pages will cause you to lose marks.

Please note submission deadline above. Late submissions will not be graded.

How to submit:

Drop your submission into a slot labeled "CS1231 Assignment" at the Undergraduate Studies Office at COM1-02-19 from 29 October 2018 onwards.

Question 1. (8 marks)

Let $A = \{a, b\}$ and S be the set of all strings over alphabet A. That is, S contains strings of the form: $a, b, abaa, bbb, babbbaabbaaaa, \ldots$, etc. Any string made up of the letters a, b (and *only* these letters) is in S. Further, we define ε as the *empty string*, ie. the string containing no letters. By definition, $\varepsilon \in S$.

Define the function $R : S \to S$ by:

For all $s \in S$, $R(s) =$ a new string that replaces the leftmost occurrence of a in s with b. If a does not occur in s, then $R(s) = s$.

Examples: $R(\varepsilon) = \varepsilon$; $R(a) = b$; $R(bbb) = bbb$; $R(baaa) = bbaa$.

Another function $C : S \to \mathbb{N}$ is defined as:

$\forall s \in S, C(s) =$ (the number of a's in s) - (the number of b's in s), or 0 if the difference is negative.

Examples: $C(bbb) = 0$; $C(aba) = 1$; $C(babaaa) = 2$.

(a) (2 marks) Determine $(R \circ R)(aba)$.

(b) (3 marks) Is R one-to-one? Prove or disprove it.

(c) (3 marks) Is C onto? Prove or disprove it.

Question 2. (4 marks)

(a) (2 marks) Aiken rolls a fair six-sided die. If a prime number is rolled, she wins an amount equal to the number rolled times \$2. If a prime number is not rolled, she loses an amount equal to the number rolled times \$3. If Aiken rolls the die 100 times, what is the expected amount she will win or lose? Express your answer to the nearest integer. Show your workings.

(b) (2 marks) Dueet is taking a 30 multiple-choice-question test, where each question has five choices. The score of the test is the number of correct answers minus one-fourth of the number of wrong answers.

Dueet is certain that he has 20 of the answers correct. On each of the remaining 10 questions, he can eliminate two of the wrong choices. Assuming that Dueet answers all questions, what is his expected score for the test? Show your workings.

Question 3. (3 marks)

You are helping to design a game at the SoC Fun Fair. The game involves rolling some dice and counting the number of 3s obtained.

You have 3 versions of the game in mind for the players to win a prize:

(A) Rolling 6 dice and getting at least one 3.

(B) Rolling 12 dice and getting at least two 3s.

(C) Rolling 18 dice and getting at least three 3s.

All dice are fair. Let p be the probability of rolling a 3. Work out the probability of each of the three versions. Write your answers in 4 significant digits.

Important: Keep your workings short. You just need to show the key steps, not all the details. Work out the details on rough papers and transfer only the essential steps to your final solution. As a guide, the answers for this question should not take up more than one page of A4 paper, or marks will be deducted.

Optional: (This part will not be graded) Just for curiosity, which version did you expect to give the players the highest chance of winning before you started working out the answers?

Question 4. (2 marks)

Prove that if you pick any five distinct points inside an equilateral triangle with side 10 cm, there is always a pair of points that are at most 5 cm apart. Your must include an appropriate diagram in your answer.

You may assume without proof that two points in an equilateral triangle of side x cm is at most x cm apart.

Question 5. (3 marks)

In a town, it is raining 1/4 of the days. If it is rainy, there will be heavy traffic with probability of 1/3; if it is not rainy, there will be heavy traffic with probability of 1/5. If it is rainy and there is heavy traffic, I arrive late for work with probability of 1/2. On the other hand, if it is not rainy and there is no heavy traffic, I will be late for work with probability of 1/10. In other situations (rainy and no heavy traffic, not rainy and heavy traffic) the probability of being late is 1/4.

You pick a random day.

(a) What is the probability that it is not raining and there is heavy traffic and I am not late?

(b) What is the probability that I am late?

(c) Given that I arrived late at work, what is the probability that it rained that day?

Important: Write your answers as fractions. Your workings should not exceed one A4 page.

Reminder: Have you written your **name, student number and tutorial group** at the top of the first page? Your tutorial group is important as we will be passing your script to your tutor for grading.

—— **END** ——