Singapore Superstar: IdolRank

(The following is based on Google's *PageRank*, a system invented by Google founders Larry Page and Sergey Brin for ranking webpages. For details on PageRank, you may google (what else!) it.)

In a *Singapore Superstar* contest where contestants compete to be the latest teen idol, the judges would like to rank the contestants based on their popularity. A measure, called **IdolRank**, was formulated, which judges a contestant's popularity based on the number of people (who are fellow contestants) who are willing to be referees for that contestant. Also, every time a person becomes somebody's referee, he/she passes some of his/her popularity (i.e. *IdolRank* value) to the person he/she refereed. The popularity computation is then computed over several iterations until it *converges* (i.e. the popularity of a person does not change further between two consecutive iterations).

The implementation details of IdolRank are as follows:

1. Every idol wannabe starts with a popularity of 1.
2. In each iteration, each contestant passes his/her current popularity value *V* to all the other contestants whom he/she refereed. Each of them gets a popularity share of *V*/*N*, where *N* denotes the number of persons whom he/she refereed.

   For example, if Person A currently has a popularity of 1, and there are 3 persons B, C and D who have Person A as their referee, then B, C and D each gets a popularity of 1/3.
3. A contestant accumulates his/her popularity from all the people who are willing to be his/her referees.
4. The process iterates until all the popularity values converge. **For simplicity, in this task, you will need to iterate for only 6 times.**

In order to capture the referee information, we make use of an *adjacency matrix*, **A**. In matrix **A**, each cell of the matrix indicates whether a person is a referee for another person. Whenever a person P1 becomes the referee for another person P2, we put 1 into the cell (P2, P1). All other cells are marked with a zero. Consider the following example:

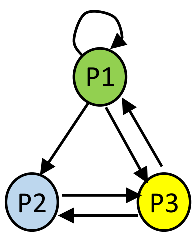

Figure 1. The relationships among three contestants

$$\begin{array}{c} \\ P1 \\ P2 \\ P3 \end{array} \begin{array}{c} \begin{array}{ccc} P1 & P2 & P3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

Figure 2. Adjacency Matrix **A**

$$\begin{array}{c} \\ P1 \\ P2 \\ P3 \end{array} \begin{array}{c} \begin{array}{ccc} P1 & P2 & P3 \end{array} \\ \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 1 & 0 \end{bmatrix} \end{array}$$

Figure 3. Popularity Distribution Matrix, PD

In Figure 1, we see the relationship among three persons: P1, P2 and P3. Person P2 is the referee for P3, thus, there is an arrow from P2 to P3 in the figure, and in the corresponding adjacency matrix **A** in Figure 2, we have a 1 in the cell (P3, P2). Similarly, since P1 is the referee for P1, P2 and P3, we see a 1 in (P1, P1), (P2, P1) and (P3, P1).

Next, if we look at the column P1 in **A**, we observe that P1 is the referee for 3 persons. Therefore, we distribute the popularity of P1 (i.e. V=1, N=3) to P1, P2 and P3. Each one of them gets 1/3. This is illustrated in Figure 3. In the first column, we can see that each one of P1, P2 and P3 gets a popularity of 1/3 from P1. In the second column, since P2 is the referee for P3 only, P3 gets all the popularity value of 1 from P2.

We represent the popularity of everyone in a single column matrix (a one-dimensional array), **P**. Using the Popularity Distribution Matrix, **PD**, the popularity of everyone is then iteratively computed as follows:

$$P_{R+1} = PD \times P_R$$

where $\mathbf{P}_R$ indicates the array **P** at the R^{th} round (iteration) and $\mathbf{P}_{R+1}$ the array **P** at the $(R+1)^{th}$ round (iteration).

Since the popularity of everyone is initially assigned to 1, the initial matrix **P** at round 0 is

$$\begin{matrix} P1 \\ P2 \\ P3 \end{matrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{R0}$$

Using the above example, the step-by-step workings for computing the **IdolRank** value till the sixth iteration is illustrated below:

$$\begin{bmatrix} \frac{5}{6} \\ \frac{5}{6} \\ \frac{4}{3} \end{bmatrix}_{R1} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{R0} \qquad \begin{bmatrix} \frac{149}{162} \\ \frac{149}{162} \\ \frac{94}{81} \end{bmatrix}_{R4} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{47}{54} \\ \frac{47}{54} \\ \frac{34}{27} \end{bmatrix}_{R3}$$

$$\begin{bmatrix} \frac{17}{18} \\ \frac{17}{18} \\ \frac{10}{9} \end{bmatrix}_{R2} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{5}{6} \\ \frac{5}{6} \\ \frac{4}{3} \end{bmatrix}_{R1} \qquad \begin{bmatrix} \frac{431}{486} \\ \frac{431}{486} \\ \frac{298}{243} \end{bmatrix}_{R5} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{149}{162} \\ \frac{149}{162} \\ \frac{94}{81} \end{bmatrix}_{R4}$$

$$\begin{bmatrix} \frac{47}{54} \\ \frac{47}{54} \\ \frac{34}{27} \end{bmatrix}_{R3} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{17}{18} \\ \frac{17}{18} \\ \frac{10}{9} \end{bmatrix}_{R2} \qquad \begin{bmatrix} \frac{1325}{1458} \\ \frac{1325}{1458} \\ \frac{862}{729} \end{bmatrix}_{R6} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{431}{486} \\ \frac{431}{486} \\ \frac{298}{243} \end{bmatrix}_{R5}$$

Figure 4. Computation of **P**

Using **P**, after the sixth iteration, we can observe that the most popular person is therefore P3 (with a popularity value of 862/729 or 1.18).

In this task, you will take as input the number of persons to be evaluated, and the adjacency matrix. There are 2 possible outputs:

a. If one of the persons has the highest popularity score, output:
   `Most popular contestant: Px`
   where *x* denotes the contestant number.

b. If **P** is a zero matrix (i.e. the popularity scores for all contestants are zeroes), output:
   `Most popular contestant: None`

Sample runs:

Sample run using interactive input (output shown in bold purple):

```
Enter the number of persons: 3
Enter the adjacency matrix : 1 0 1 0 0 1 1 1 0

Most popular contestant: P1
```

Another sample run

```
Enter the number of persons: 3
Enter the adjacency matrix : 0 1 1 0 1 0 1 0 1

Most popular contestant: P3
```

Notes:

- If there are more than one most popular person (with the same popularity score), output only the largest contestant number.
- There are at most 5 contestants.
- Can you think of instances where the technique **IdolRank** would not work?

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