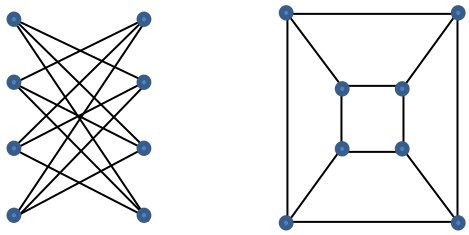
Tutorial 11
Graphs II and Trees

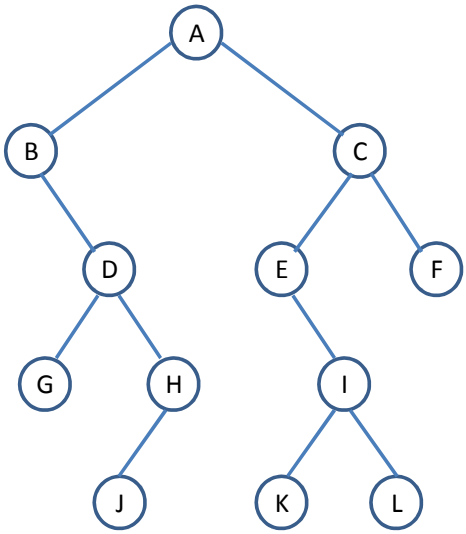
This is the final tutorial! Yeah!
See you at the CS1231 exam on 1 December! All the best for your exams!—

1 Discussion questions

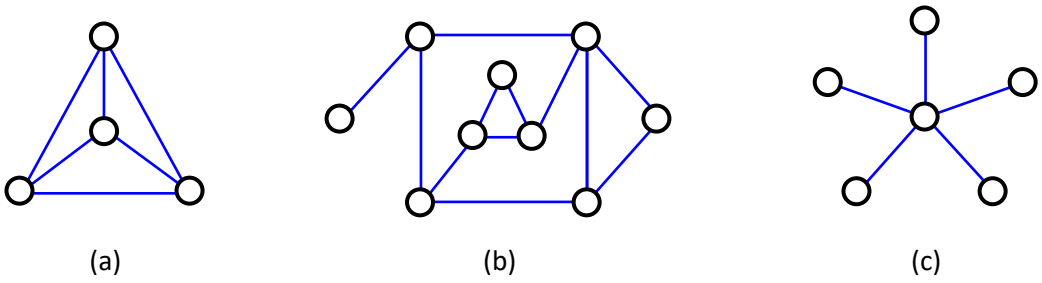
D1. The two graphs G_1 and G_2 below are isomorphic. Assign labels $1, 2, ..., 8$ to the vertices to show that the two graphs have the same edge set.



D2. Given the following binary tree, write out the pre-order, in-order, and post-order traversals of its vertices.



D3. Verify Euler's formula $f = e - v + 2$ on the planar graphs below.



(a) (b) (c)

2 Tutorial questions

Q1. A *self-complementary* graph is isomorphic with its complement. Draw a self-complementary graph with (a) four vertices; (b) five vertices.

Q2. Prove Euler's formula $f = e - v + 2$ for any connected simple planar graph by using induction on the number of edges in the graph.

Q3. Construct the binary tree given the following in-order and pre-order traversals of the tree:

- In-order: I A D J N H B E K O F L G C M
- Pre-order: H N A I J D O B K E C L F G M

Draw diagrams to trace the steps of your construction.

Q4. We have proved Lemma 10.5.1: that any non-trivial tree has at least one vertex of degree one.

Now, prove that any non-trivial tree actually has at least two vertices of degree one, which was claimed in class without proof.

In your proof, make use of the theorem(s) covered in class and arithmetic, instead of using the visualisation proof that was used for Lemma 10.5.1.

Q5. (a) Draw a simple graph that has a Hamiltonian circuit and an Euler circuit.

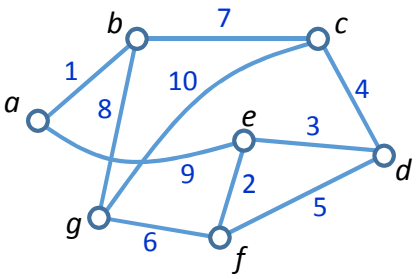
(b) Draw a simple graph that has a Hamiltonian circuit but no Euler circuit.

(c) Draw a simple graph that has an Euler circuit but no Hamiltonian circuit.

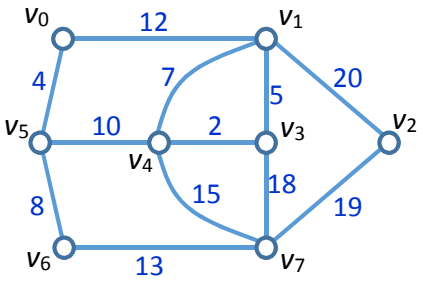
(d) Draw a simple graph that has neither a Hamiltonian circuit nor an Euler circuit.

Q6. Find all non-isomorphic trees with five vertices.

Q7. Use Kruskal's algorithm to find a minimum spanning tree (MST) for each of these graphs, giving the total weight of the MST. Indicate the order in which edges are added to form each tree.



(a)



(b)

Q8. Use Prim's algorithm starting with vertex a or v_0 to find a minimum spanning tree (MST) for each of the graphs in question 7 above. Indicate the order in which edges are added to form each tree.