Tutorial 6
Sets and Relations

1 Discussion questions

Discussion questions are meant for discussion on the IVLE Forum. You may try them on your own or discuss them with your classmates. No answers will be provided by us.

D1. Compute the following sets.

(a) $\bigcup\{\,\{1\},\ \{1,2\},\ \{1,2,3\}\,\}$.
(b) $\{1,2,4\} \cap \{1,3,2\}$.
(c) $\{1,2,4\} \ominus \{1,3,2\}$.

D2. Partitions. For each set listed below, explain whether or not it is a partition of $\{\,1,2,3,4\,\}$.

(a) $\{\,\{1\},\ \{2,3,4\}\,\}$.
(b) $\{\,\{1\},\ \{3,4\},\ \{\}\,\}$.
(c) $\{\,\{2\},\ \{3,4\},\ \{1,2\}\,\}$.

D3. Determine all the possible partitions of $\{\,a,b,c\,\}$. How many are there?

2 Tutorial questions

Q1. Write the correct symbol to replace the ? in the following expressions. Choose from $\in$, $\subseteq$, $\notin$ and $\not\subseteq$.

(a) $\{1,2\}$? $\{1,3,2\}$.
(b) 1 ? $\{1,3,2\}$.
(c) 0 ? $\{1,3,2\}$.
(d) $\varnothing$? $\{1,3,2\}$.
(e) $\{1,2\}$? $\{1,3,2,\ \{1,2\}\}$.

Q2. Power sets. Determine these sets:

i. $\mathcal{P}(\{1,2,4\})$.
ii. $\mathcal{P}(\mathcal{P}(\mathcal{P}(\varnothing)))$.

Q3. Define $A_n = \left\{\ x \in \mathbb{R} \mid \frac{-1}{n} < x < 2 - \frac{1}{n}\ \right\}$, for any $n \in \mathbb{Z}^+$.

Determine the following sets:

(a) $\bigcup_{n\in\mathbb{Z}^+} A_n$, that is, $A_1 \cup A_2 \cup A_3 \cup \cdots$.
(b) $\bigcap_{n\in\mathbb{Z}^+} A_n$, that is, $A_1 \cap A_2 \cap A_3 \cap \cdots$.

Q4. Prove or disprove this statement:

$$\text{For all sets } A \text{ and } B,\ \text{if } A \subseteq B \text{ then } A \cap B^c = \varnothing.$$

Q5. Let $A = \{5, 6, 7\}$ and $B = \{apple, apricot, durian, grape, orange, peach, pumpkin\}$. Define a binary relation $\mathcal{L}$ from A to B as follows:

$$\forall a \in A, \forall b \in B,\ a\ \mathcal{L}\ b\ \leftrightarrow a \text{ is the length of } b.$$

(a) Draw the arrow diagram for $\mathcal{L}$, and list its elements.

(b) Draw the arrow diagram for $\mathcal{L}^{-1}$, and list its elements.

Q6. For the binary relation $\mathcal{R}$ shown in Figure 1, check if it is reflexive, symmetric, and transitive. If $\mathcal{R}$ possesses each of these properties, explain why. If not, fill in the minimum number of arrows so that $\mathcal{R}$ will have that property.

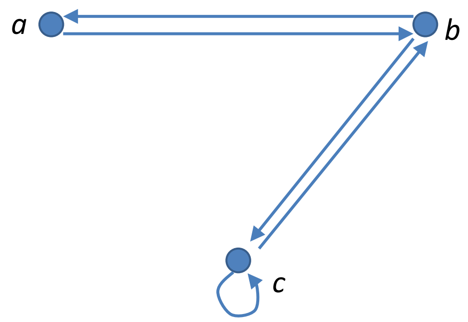

Figure 1: Diagram showing a binary relation $\mathcal{R}$.

Q7. For the binary relation $\mathcal{R}$ shown in Figure 1, compute:

(a) $\mathcal{R} \circ \mathcal{R}$

(b) $\mathcal{R} \circ \mathcal{R} \circ \mathcal{R}$

(c) $(\mathcal{R} \circ \mathcal{R}) \cup \mathcal{R}$

Which of the above is transitive?

Q8. Let $\mathcal{R}$ and $\mathcal{S}$ be two relations defined on the same set A. Prove or disprove each statement:

(a) If both $\mathcal{R}$ and $\mathcal{S}$ are symmetric, then $\mathcal{R} \cap \mathcal{S}$ is symmetric.

(b) If both $\mathcal{R}$ and $\mathcal{S}$ are transitive, then $\mathcal{R} \cup \mathcal{S}$ is transitive.

Note that relations are, in fact, subsets of Cartesian Products, which means we can perform set operations on them.

Q9. Define a relation $\sim$ on $\mathbb{Z} - \{0\}$ as follows: $\forall a, b \in \mathbb{Z} - \{0\},\ a \sim b$ iff $ab > 0$.

(a) Prove that $\sim$ is an equivalence relation.

(b) Determine all the distinct equivalence classes formed by this relation.