

## CS1231S Assignment #1

AY2024/25 Semester 1

**Deadline: Monday, 16 September 2024, 1:00pm**

**IMPORTANT: Please read the instructions below carefully.**

This is a graded assignment worth 10% of your final grade. There are **six questions** (an admin Q0 and six task questions Q1–6) with a total score of 40 marks. Please work on it by yourself, not in a group or discussion/in collaboration with anybody. Anyone found committing plagiarism (submitting other’s work as your own), or sending your answers to others, or other forms of academic dishonesty, will be penalised with a straight zero for the assignment, and reported to the school. Please see SoC website “Preventing Plagiarism” <https://www.comp.nus.edu.sg/cug/plagiarism/>.

You must submit your assignment to **Canvas > Assignments > Assignment 1 submission** before the deadline. **Late submission will NOT be accepted**. We have set the closing time of submission to slightly after 1pm to give you a few minutes of grace, but in your mind, you should treat **1pm** as the deadline. If you think you might be too busy on the day of the deadline, please submit earlier. Also, avoid submitting in the last few minutes, as the system may get sluggish due to overload and you will miss the deadline, and we will not extend the deadline for you.

The question paper is on **Canvas > Files > Assignment 1**. In the same folder you can find the template submission files **assign1\_template.docx** and **assign1\_template.pdf**. Download one of these template files. You may proceed in one of these ways: type your answers in assign1\_template.docx and convert it into a pdf file, or use a pdf editor and type your answers directly into assign1\_template.pdf. You must rename the pdf file to **Axxxxxxx.pdf** before you submit it, where Axxxxxxx is your student number.

Note that we are using a software to scan your shaded Student Number, so the pdf file you submit must have the Student Number box (refer to the template file) at the exact position. Also, the answers you write must fall within the boxes at those fixed locations in the template file. So, **do not use your phone camera** to scan your file as it will likely not have the same pagination and positioning as the template files we provide.

You are to submit a **SINGLE pdf file**. You may submit multiple times, but only the last submitted file will be graded.

Note the following:

- Shade your **Student Number** correctly, as instructed on the template file.
- As this is an assignment given well ahead of time, we expect you to work on it early. You should submit **polished work**, not answers that are untidy or appear to have been done in a hurry, for example, with scribbling and cancellation all over the places. Marks may be deducted for untidy work.
- It is always good to be clear and leave no gap so that the marker does not need to make guesswork on your answer. When in doubt, the marker would usually not award the mark.
- Write/type your answers within the boxes given in the template file.
- Do not use any methods that have not been covered in class. When using a theorem or result that has appeared in class (lectures or tutorials), please quote the theorem number/name, the lecture and slide number, or the tutorial number and question number in that tutorial, failing which marks may be deducted. Remember to use numbering and give justification for important steps in your proof, or marks may be deducted.
- **[Very important!]** In the past, some students submitted a single-page file or even a blank file, or did not submit anything, without realising it! It is your responsibility to check that your submitted file is complete. We will not allow any submission after the deadline.

If you need any clarification about this assignment, please do NOT email us or post on telegram, but post on **QnA “Assignments”** topic so that everybody can read the answers to the queries.

**Question 0.** (Total: 2 marks)

Check that ...

- you have submitted a pdf file with your Student Number as the filename. [1 mark]
- you have written both your name and tutorial group number (eg: T02) at the top of the first page of your file. (If you miss either one you will not be given any mark.) [1 mark]

**Question 1.** Propositional logic (Total: 7 marks)

(a) Prove the following statements using truth table for part (i) and Theorem 2.1.1 for part (ii). For truth tables, you may use **T** for true and **F** for false. For Theorem 2.1.1, you must write **true** and **false** for tautology and contradiction respectively.

(i)  $p \wedge (\sim p \vee q) \equiv p \wedge q$  [1 mark]

(ii)  $p \vee (\sim p \wedge q) \equiv p \vee q$  [2 marks]

The above are variants of the absorption laws in Theorem 2.1.1. To distinguish them from the absorption laws, we shall call them the “variant absorption laws” and you may cite them in your work from now on.

(b) Simplify the statement form below in no more than 9 steps. Make sure you do not skip any step, and every step must be justified by a law. Do not combine two steps of the same law in a single step. Use **true** and **false** for tautology and contradiction respectively. (This question checks that you apply the laws rigorously and cite them correctly, so we will be strict in our grading.) [4 marks]

$$(p \wedge (p \rightarrow r \vee q)) \wedge (r \rightarrow q)$$

**Question 2.** Argument (Total: 6 marks)

Determine whether the following argument is valid or invalid, assuming that  $p, q, r$  and  $s$  are statement variables:

$$(p \wedge q) \rightarrow r$$

$$(r \vee s) \rightarrow p$$

$$r \wedge \sim p$$

$$\therefore s \vee q$$

You should begin your answer by stating “Valid” or “Invalid”, followed by your proof. You are not allowed to show your truth table (partial or full) in your answer, though you may use truth table in your own rough work to help you derive your proof.

**Question 3.** Sets (Total: 6 marks)

No working/justification is required for this question.

(a) For each  $i \in \{1,2,3,4\}$ , define  $A_i = \{ik + 1 : k \in \{0,1,2,3,4\}\}$ . Write down  $B_2, B_3, B_4$  in **roster notation** with the following properties, where  $B_1, B_2, B_3, B_4$  are mutually disjoint sets:

$$B_1 = A_1,$$

$$B_1 \cup B_2 = A_1 \cup A_2,$$

$$B_1 \cup B_2 \cup B_3 = A_1 \cup A_2 \cup A_3,$$

$$B_1 \cup B_2 \cup B_3 \cup B_4 = A_1 \cup A_2 \cup A_3 \cup A_4.$$

Do not use ellipses (“...”) in your answer. [3 marks]

(b) For each  $k \in \mathbb{Z}_{\geq 0}$ , let  $D_k = \{n \in \mathbb{Z}_{\geq 0} : k = mn \text{ for some } m \in \mathbb{Z}_{\geq 3}\}$ . Write down each of the following sets in **roster notation**. [3 marks]

(i)  $D_2$

(ii)  $D_3$

(iii)  $D_6$

(iv)  $D_{18}$

(v)  $\bigcap_{k=6}^{14} D_k$

(vi)  $\bigcup_{k=6}^{14} D_k$

**Question 4.** Quantified statements on Sets (Total: 6 marks)

Let  $A = \{-2, -1, 0, 1, 2\}$ ,  $B = \{0, 1, 4\}$  and  $C = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ .

For each of the parts below, determine whether the given statement is true or false. Start your answer by writing "True" or "False", followed by your proof or counterexample. The symbol  $\Leftrightarrow$  is the biconditional connective representing "if and only if".

- (a)  $\forall x \in C ((x \in A) \Leftrightarrow (x^2 \in B))$ . [3 marks]
- (b)  $\exists x \in A \forall y \in A ((x \neq 0) \wedge (xy \in B))$ . [3 marks]

**Question 5.** Proof (Total: 7 marks)

Consider this statement:

For any integers  $x$  and  $y$ , if  $x + y$  is even, then both  $x$  and  $y$  are even or both are odd.

The following is Aiken's proof:

"We prove by contradiction. Suppose  $x + y$  is even and for  $x$  and  $y$ , one of them is even and the other is odd. Without loss of generality, we may assume that  $x$  is even and  $y$  is odd. Then,  $x = 2k$  for some integer  $k$  (by definition of even number) and  $y = 2m + 1$  for some integer  $m$  (by definition of odd number). So we have  $x + y = 2k + 2m + 1 = 2(k + m) + 1$ . Since  $k + m$  is an integer (by closure of integers under addition),  $x + y$  is therefore odd. This contradicts the assumption that  $x + y$  is even."

In your answer, you may use the predicates  $Even(n)$  to mean " $n$  is even" and  $Odd(n)$  to mean " $n$  is odd". Also recall in Lecture 1 slide 27 Assumption 1: Every integer is even or odd, but not both. This means  $\sim Even(n) \equiv Odd(n)$ .

- (a) Rewrite the given statement using mathematical notation. Your answer should not contain any English words like "such that". [2 marks]
- (b) Explain why the proof begins by assuming that one integer is even and the other is odd. You may skip the commutative laws in your explanation, and you may combine the same rule in consecutive steps into one step (eg: negation law twice). (Your explanation should be in a form of a proof, rather than an English essay.) [4 marks]
- (c) Explain what is meant by "without loss of generality" in this proof. [1 mark]

**Question 6.** Proof on Sets (6 marks)

Prove or disprove the statements below. Do not use Venn diagrams. Start your answer by writing "True" or "False", followed by your proof or counterexample.

- (a) For all sets  $A, B$  and  $C$ ,  $(A \cap B) \times (B \cap C) = (B \cap C) \times (A \cap B)$ . [1 mark]
- (b) For every set  $A$ , if  $A \subseteq \emptyset$ , then  $A = \emptyset$ . [2 marks]
- (c) For all sets  $A$  and  $B$ ,  $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$ . (Note:  $\mathcal{P}(X)$  is the power set of  $X$ .) [3 marks]

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