## NATIONAL UNIVERSITY OF SINGAPORE

## CS1231S - DISCRETE STRUCTURES

(Semester 2: AY2022/23)
Final Assessment Answer Sheet
Time Allowed: 2 Hours

## INSTRUCTIONS

1. Write your Student Number on the right AND, using pen or pencil, shade the corresponding circle completely in the grid for each digit or letter. DO NOT WRITE YOUR NAME!
2. Zero mark will be given if you write/shade your Student Number incompletely or incorrectly.
3. Write your Student Number at the top of pages 3 and 5.
4. This answer sheet comprises SIX (6) pages.
5. All questions must be answered in the space provided; no extra sheets will be accepted as answers.
6. You must submit only this ANSWER SHEET and no other documents.

四STUDENT NUMBER

7. An excerpt of the question may be provided to aid you in answering in the correct box. It is not the exact question. You should still refer to the original question in the question paper.
8. You may write your answers using pencil (at least 2B) or pen as long as it is legible (no red ink, please).
9. The maximum mark for this paper is 100 .
10. Marks may be deducted for (i) illegible handwriting, and/or (ii) excessively long explanations.
11. Each multiple choice question is intended to have only one answer. Shade the appropriate bubbles using pencil only.

| For Examiner's Use Only |  |  |
| :---: | :---: | :---: |
| Question | Marks | Remarks |
| Q1-20 | $/ 40$ |  |
| Q21 | $/ 5$ |  |
| Q22 | $/ 20$ |  |
| Q23 | $/ 20$ |  |
| Q24 | $/ 9$ |  |
| Q25 | $/ 100$ |  |
| Total |  |  |

## Part A: Multiple Choice Questions (Total: 40 marks)

Please shade only ONE bubble for each question. Please use ONLY pencil to shade.

|  | $(A)$ | $(B)$ | $(C)$ | $(D)$ | $(E)$ |
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| 1. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
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| 3. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 4. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 5. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 6. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
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| 9. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 10. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 11. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 12. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 13. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 14. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 15. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 16. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 17. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 18. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 19. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
| 20. | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ | $\bigcirc$ |
|  | $(A)$ | $(B)$ | $(C)$ | $(D)$ | $(E)$ |

Part B (Total: 60 marks)
21. Mathematical induction. [ 5 marks]

1. For each $n \in \mathbb{N}$, let $P(n) \equiv\left(\operatorname{Even}\left(F_{n}\right) \Leftrightarrow \operatorname{Even}\left(F_{n+3}\right)\right)$.
2. Graphs and trees. [20 marks]
(a) Which of $C_{4}, S_{4}, W_{4}$ are complete graphs?
(b) Which of $C_{1231}, S_{1231}, W_{1231}$ are planar graphs?
(c) $C_{1231}$ has an Euler circuit. True or false?
$W_{1231}$ has an Euler circuit. True or false?
d)
$\square$ colours for $C_{5}$; $\square$ colours for $S_{5}$; $\square$ colours for $W_{5}$.
3. Graphs and trees. (continue...) [20 marks]
(e) All non-isomorphic spanning trees of $C_{5}, S_{5}, W_{5}$.
(f) Possibility tree to determine the number of Hamiltonian paths in $W_{4}$ with initial vertex at $v_{1}$.
(g) Number of unique open-loop Hamiltonian paths in $W_{4}$ :


(i) Height of binary tree is
(j) $\left\{V_{1}, V_{2}, \cdots, V_{k}\right\}$ is always a partition of $V$. True or false?
$\left\{E_{1}, E_{2}, \cdots, E_{k}\right\}$ is always a partition of $E$. True or false?

4. Counting and probability. [20 marks]
(a)
[1]

(d) $\square$
(e)
(i)
$[1]$
${ }_{[1]}^{\text {(ii) }} P(X=2)=$
$\int_{[3]}^{\text {(iii) }} E(X)=$
(f)
(i) $\left|S_{1 v}\right|$ [2]
(f)
(iii) $\left|S_{\geq 1 v}\right|$ [2]
(iv) $\left|S_{\geq 2 v}\right|$
[2]
5. Relations and functions. [9 marks]
$\square$
6. Relations and functions. (continue...) [9 marks]
(c)
[3]
Order of $f=\square ; \quad$ order of $g=\square ; \quad$ order of $\left(f^{-1} \circ g\right)=\square \square$ 25. Cardinality. [6 marks]
$\square$
