CS1231S Revision

14 November 2024

AY2023/24 Semester 2
Exam Paper

MCQs

1	C	2	A	3	A	4	B	5	A
6	B	7	D	8	B	9	B	10	D
11	B	12	B	13	E	14	D	15	B
16	D	17	A	18	E	19	B	20	B
21	A	22	C	23	D	24	C		

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Hardest MCQ

4. Consider any set A and predicate $P(x, y)$, which of the following statements are true?

(i) $\forall x \in A\ \forall y \in A\ P(x, y) \rightarrow \exists x \in A\ \exists y \in A\ P(x, y)$.

(ii) $\exists x \in A\ \exists y \in A\ P(x, y) \rightarrow \forall x \in A\ \forall y \in A\ P(x, y)$.

(iii) $\forall x \in A\ \exists y \in A\ P(x, y) \rightarrow \exists y \in A\ \forall x \in A\ P(x, y)$.

(iv) $\exists x \in A\ \forall y \in A\ P(x, y) \rightarrow \forall y \in A\ \exists x \in A\ P(x, y)$.

A. Only (i).

B. Only (iv).

C. Only (i) and (iv).

D. All of (i), (ii), (iii) and (iv).

E. None of options (A), (B), (C), (D) is correct.

Counterexamples: (i) $A = \emptyset$;
(ii) and (iii) $A = \mathbb{Z}, P(x, y) \equiv y > x$.
(iv) If $A = \emptyset$, then statement is vacuously true.
If $A \neq \emptyset$, let $a \in A$ such that $P(a, y)\ \forall y \in A$.

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18. Suppose A and B are finite sets and f and g are functions, which of the following statements are true?

(i) If $|A| > |B|$, then there is no surjection from A to B by the pigeonhole principle.

(ii) If g and $g \circ f$ are surjections, then f must be a surjection.

(iii) If $g \circ f$ is an identity function, then f or g is an identity function, or both f and g are identity functions.

(i), (ii), (iii) are false.

(ii) Counter-example: $A = \{a\}, B = \{p, q\}, C = \{z\}$.
Let $f : A \to B$ and $g : B \to C$ where $f(a) = p$ and $g(p) = g(q) = z$.
So, g and $g \circ f$ are surjections, but f is not.

A. Only (i).
B. Only (ii).
C. Only (i) and (ii).
D. Only (ii) and (iii).
E. None of options (A), (B), (C), (D) is correct.

25. R s a non-empty relation on $A = \{a, b\}$. Let S be any relation on A where $R \subseteq S$.

(a) If R is reflexive, then S is reflexive. **True**

(b) If R is irreflexive, then S is irreflexive. **False** $R = \{(a,b)\}, S = \{(a,a),(a,b)\}$.

(c) If R is symmetric, then S is symmetric. **False** $R = \{(a,a)\}, S = \{(a,a),(a,b)\}$.

(d) If R is transitive, then S is transitive. **False** $R = \{(a,b)\}, S = \{(a,b),(b,a)\}$.

(e) If R is antisymmetric, then S is antisymmetric. **False** Same counterexample as (d).

(f) If R is asymmetric, then S is asymmetric. **False** Same counterexample as (d).

Let R be a binary relation on a set A.
R is irreflexive if, and only if, $\forall x \in A\ (x, x) \notin R$.

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26. Let $n \in \mathbb{Z}$ and $d \in \mathbb{Z}^+$. Suppose $n = dq + r$ and $0 \le r < d$, we define $n \ \%\ d = r$.

Given a function $f(x)$, we define the function $\boldsymbol{f^{(n)}(x)}$ to be the result of n applications of f to x, where $n \in \mathbb{Z}^+$. For example, $f^{(3)}(x) = f\Big(f\big(f(x)\big)\Big)$. We also define the ***order*** of an input x with respect to f to be the smallest positive integer m such that $f^{(m)}(x) = x$.

Define a function $g : A \to A$ by setting, for each $x \in A$, $g(x) = 3x \ \%\ 5$.

(a) What is $g^{(3)}(21)$?

$g^{(3)}(21) = g^{(2)}(3) = g(4) = \mathbf{2}$

(b) What is the order of 3 with respect to the function g?

Order of 3 is **4**.
Working: $g^{(4)}(3) = g^{(3)}(4) = g^{(2)}(2) = g(1) = 3$.

Quotient-Remainder Theorem:
Given any integer n and positive integer d, there exist unique integers q and r such that $n = dq + r$ and $0 \le r < d$.

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26. Let $n \in \mathbb{Z}$ and $d \in \mathbb{Z}^+$. Suppose $n = dq + r$ and $0 \le r < d$, we define $n \% d = r$.

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Define a function $g : A \to A$ by setting, for each $x \in A$, $g(x) = 3x \% 5$.

(c) Let $A = \{0,1,2,3,4\}$. Define the relation R on A as follows:

$x\ R\ y$ iff the order of x is equal to the order of y with respect to the function g.

R is an equivalence relation. Write out all the distinct equivalence classes of R using set-roster notation. Do not use the [ ] notation.

2 equivalence classes: $\{\mathbf{0}\}$ and $\{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$.

Order of 0 = 1.
Order of 1,2,3,4 = 4.

Quotient-Remainder Theorem:
Given any integer n and positive integer d, there exist unique integers q and r such that $n = dq + r$ and $0 \le r < d$.

27. (a) A bag contains 9 white balls and 5 black balls. What is the probability of picking two balls of different colours without replacement?

$$\left(\frac{9}{14}\times\frac{5}{13}\right)+\left(\frac{5}{14}\times\frac{9}{13}\right)=\frac{90}{182}=\mathbf{\frac{45}{91}}$$

(b) If you draw 6 cards from a standard deck of playing cards without replacement, what is the probability of drawing no more than 2 spades? Write your answer in percentage with 3 significant figures.

Number of ways to draw no more than 2 spades = $\binom{13}{0}\binom{39}{6}+\binom{13}{1}\binom{39}{5}+\binom{13}{2}\binom{39}{4}=17163042.$

Number of ways to draw 6 cards = $\binom{52}{6}=20358520.$

Answer = $\frac{17163042}{20358520}=\mathbf{84.3}\%$

27. (c) Given a loaded 6-sided die, the probability of rolling each number is in a direct ratio with its value, that is, the probability of rolling the number n is n/m times the probability of rolling the number m. What is the probability of rolling an even number? Write your answer as a single fraction?

Let the probability of rolling 1 be x. Then the probability of rolling n would be nx.

Hence, $x + 2x + 3x + 4x + 5x + 6x = 1$, or $x = 1/21$.

Hence, probability of rolling an even number = $\frac{2}{21} + \frac{4}{21} + \frac{6}{21} = \frac{12}{21} = \mathbf{\frac{4}{7}}$.

27. (d) A bulb factory has 3 machines *A*, *B* and *C*, manufacturing respectively 25%, 35% and 40% of the bulbs. Machines *A*, *B* and *C* have respectively 5%, 4% and 2% of defective output. A bulb is chosen at random and found to be defective. What is the probability that it was manufactured by machine *A*? Write your answer as a single fraction.

$P(A) = \frac{25}{100}, P(B) = \frac{35}{100}, P(C) = \frac{40}{100}$.

Let $P(D)$ be the probability that a bulb is defective.

$$P(D) = P(A) \times \frac{5}{100} + P(B) \times \frac{4}{100} + P(C) \times \frac{2}{100} = \frac{345}{10000}$$

$$P(A \cap D) = P(A) \times \frac{5}{100} = \frac{125}{10000}$$

Therefore, $P(A|D) = \frac{P(A \cap D)}{A(D)} = \frac{125}{345} = \mathbf{\frac{25}{69}}$

27. (e) Two fair 6-sided dice are tossed. The absolute difference is the difference in the numbers shown on the dice in absolute value. For example, if one die shows a 2 and the other a 5, then the absolute difference is 3. What is the expected value of the absolute difference of two dice? Write your answer as a single fraction.

$P(d=0)=\frac{6}{36}$; $P(d=1)=\frac{10}{36}$; $P(d=2)=\frac{8}{36}$; $P(d=3)=\frac{6}{36}$;

$P(d=4)=\frac{4}{36}$; $P(d=5)=\frac{2}{36}$.

Therefore, expected value = $0\cdot\frac{6}{36}+1\cdot\frac{10}{36}+2\cdot\frac{8}{36}+3\cdot\frac{6}{36}+4\cdot\frac{4}{36}+5\cdot\frac{2}{36}=\frac{70}{36}=\mathbf{\frac{35}{18}}$

27. (f) How many solutions are there for the equation below, where x, y, z are non-negative integers? Write your answer as a single integer.

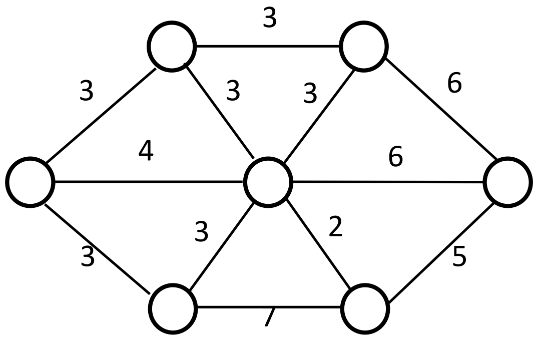
$$x + y + z = 6$$

Multiset problem with $n = 3, r = 6$: $\binom{n-1+r}{r} = \binom{8}{6} = \mathbf{28}$.

(g) How many ways can you write 1000000 (one million) as a product of 3 non-negative integers, if the order of integers in the product matters? Write your answer as a single integer. (Hint: You may make use of part (f) above.)

Note that $1000000 = 10^6 = 2^6 \cdot 5^6$. Let $1000000 = nmk$. Then $n = 2^{n_1} \cdot 5^{n_2}$, $m = 2^{m_1} \cdot 5^{m_2}$ and $k = 2^{k_1} \cdot 5^{k_2}$. Note that $n_1 + m_1 + k_1 = 6 = n_2 + m_2 + k_2$. Therefore, the answer is the square of the number of ways to write 6 as a sum of 3 non-negative integers, or $\binom{8}{2}^2 = 28^2 = \mathbf{784}$

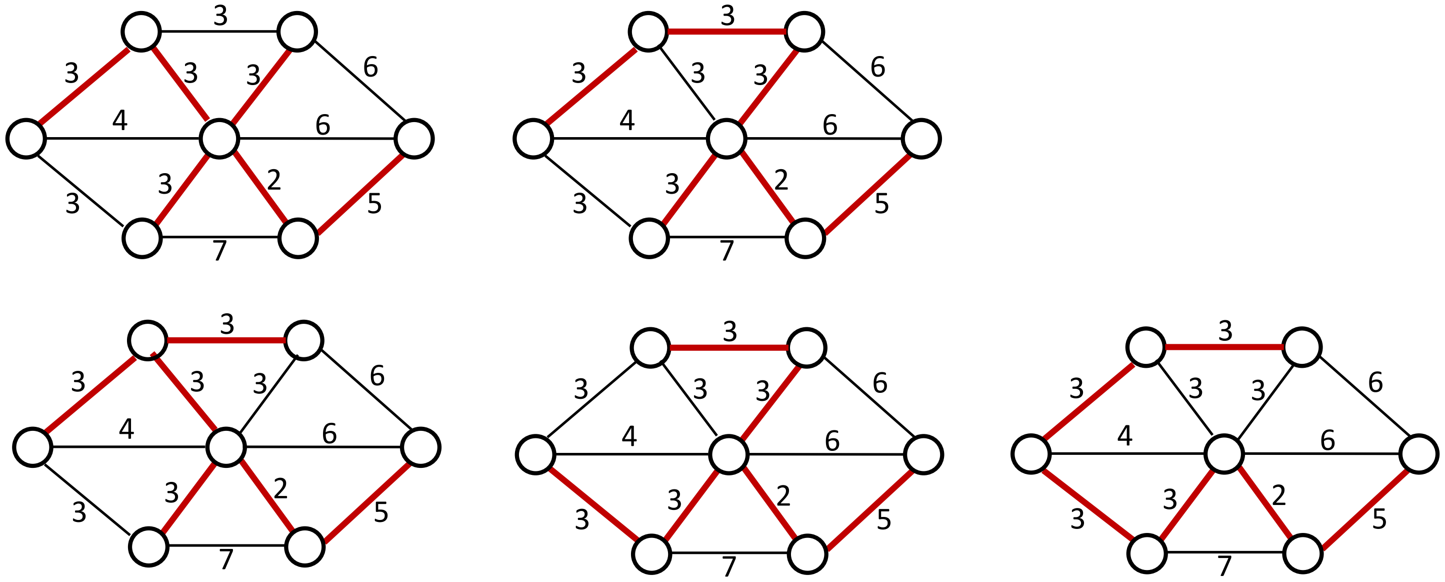
28. (a) What is the weight of the MST of the undirected weighted graph shown below?



Weight of MST is 19.

(b) Referring to the undirected weighted graph in part (a) above, how many non-isomorphic minimum spanning trees are there?

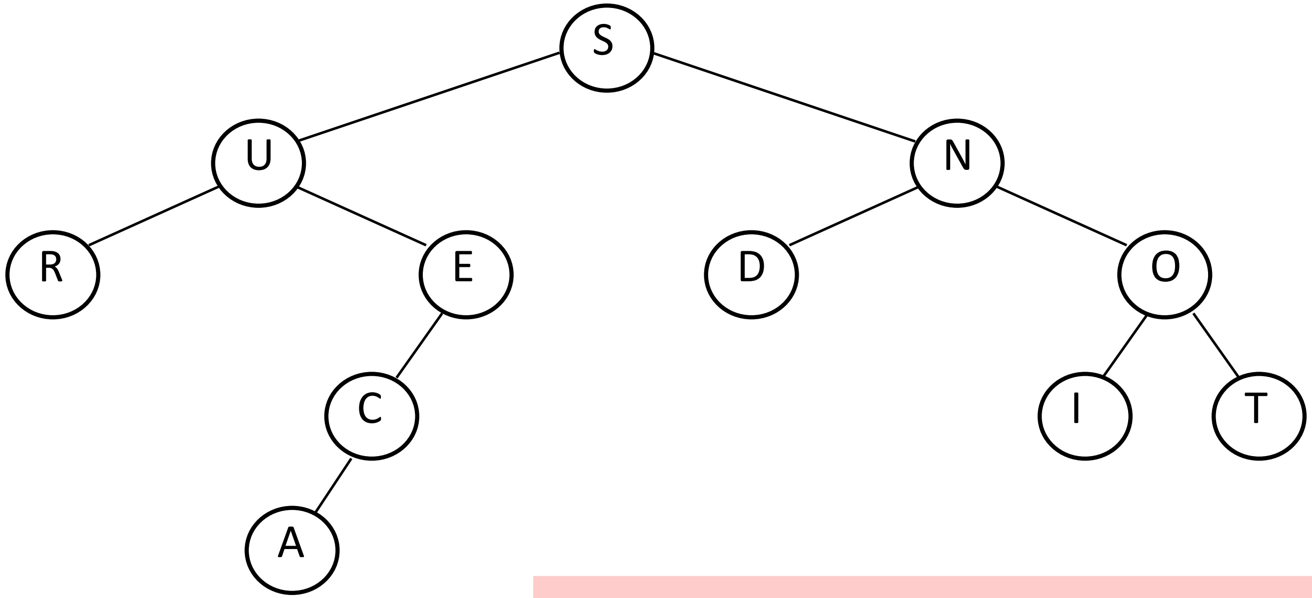
5 non-isomorphic MSTs.



28. (c) The preorder and inorder traversals of a certain binary tree are given below. Write out the postorder traversal of this tree.

Preorder: S U R E C A N D O I T

Inorder: R U A C E S D N I O T



Postorder : **R A C E U D I T O N S**

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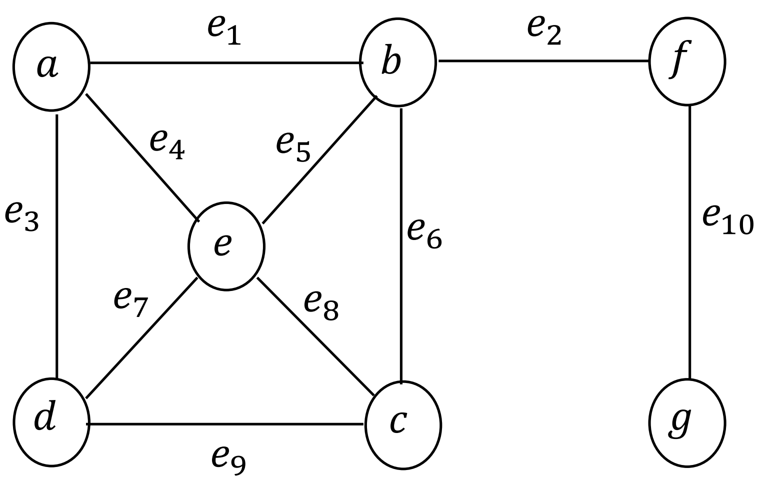
28. (d) Let G be a connected graph. A vertex is a **cut-vertex** such that removing it and all edges incident to it disconnects G, that is, increasing the number of components in G. List out any two cut-vertices in the graph below.



$\boldsymbol{b}$ and $\boldsymbol{f}$.

Cutsets with cardinality 1: $\{\boldsymbol{e_2}\}$;
cardinality 3: $\{\boldsymbol{e_1}, \boldsymbol{e_5}, \boldsymbol{e_6}\}$;
cardinality 4: $\{\boldsymbol{e_1}, \boldsymbol{e_4}, \boldsymbol{e_7}, \boldsymbol{e_9}\}$, $\{\boldsymbol{e_1}, \boldsymbol{e_5}, \boldsymbol{e_8}, \boldsymbol{e_9}\}$, $\{\boldsymbol{e_3}, \boldsymbol{e_4}, \boldsymbol{e_5}, \boldsymbol{e_6}\}$, $\{\boldsymbol{e_3}, \boldsymbol{e_7}, \boldsymbol{e_8}, \boldsymbol{e_6}\}$.

(e) Let G be a connected graph. A **cutset** S of G is a minimal set of edges of G such that the removal of all the edges in S from G disconnects G, and no proper subset of S is a cutset. List out any three cutsets in the graph in part (d) above, such that each cutset disconnects G into components with at least 2 vertices in each component. Your three cutsets must have different cardinalities.

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28. (f) Let $G = (V, E)$ be an undirected graph, and let $U \subseteq V$. The induced subgraph $G[U]$ is a graph whose vertex set is U and whose edge set consists of all the edges in E that have both endpoints in U. That is, for any two vertices $x, y \in U$, x and y are adjacent in $G[U]$ if and only if they are adjacent in G.
Figure 0 shows a graph G and Figures 1-5 show five subgraphs of G, namely, G_1, G_2, G_3, G_4 and G_5. Which of G_1, G_2, G_3, G_4 and G_5 are induced subgraphs of G?

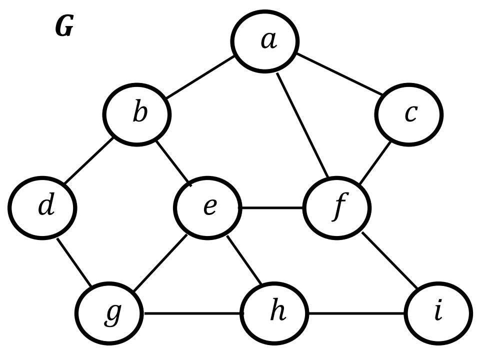

Figure 0

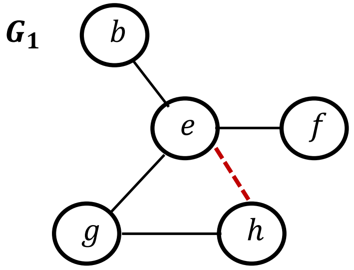

Figure 1

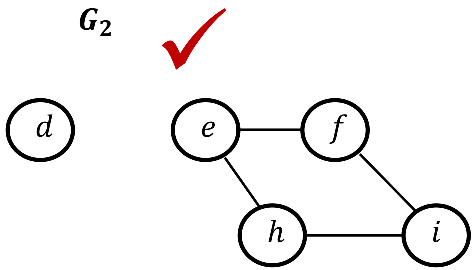

Figure 2

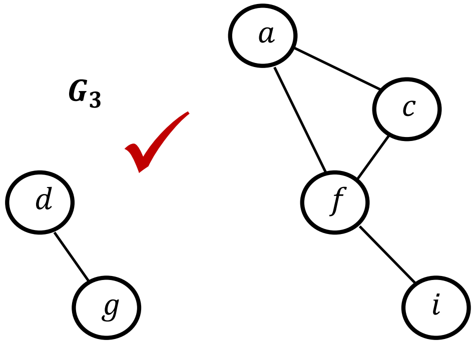

Figure 3

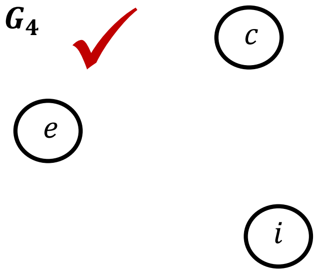

Figure 4

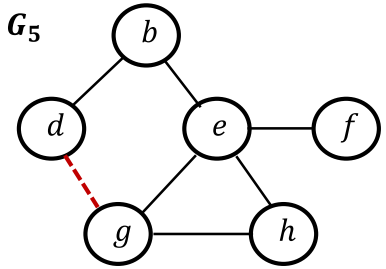

Figure 5

28. (g) In a country with 30 cities, every city is connected to every other city with a two-way road. At most how many roads can be closed such that you can still get from any city to another city?

A complete graph K_n has $\binom{n}{2}$ edges. The minimum number of edges a graph with n vertices must have in order for the graph to be connected is $n-1$.

Therefore, the maximum number of edges that can be removed from K_n so that the resulting graph is still connected is $\binom{n}{2}-(n-1)$.

For $n=30$, we have $\binom{30}{2}-(30-1)=435-29=\mathbf{406}$

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Exam Paper

MCQs

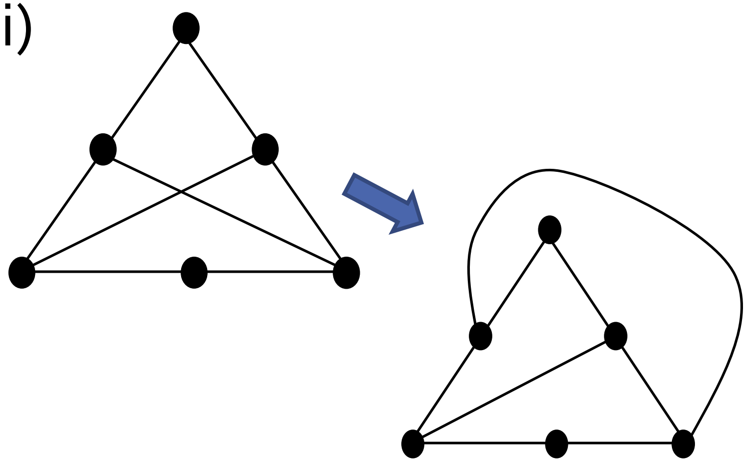
1	A	2	C	3	C	4	C	5	A
6	B	7	C	8	D	9	B	10	E
11	B	12	C	13	D	14	A/C/D	15	C
16	A	17	C	18	C	19	D	20	C
21	C	22	B						

Q14 was incorrectly set. There are 3 correct answers.

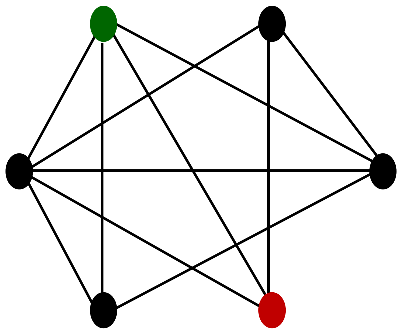
Hardest MCQ

17. Which of the following graphs are planar?

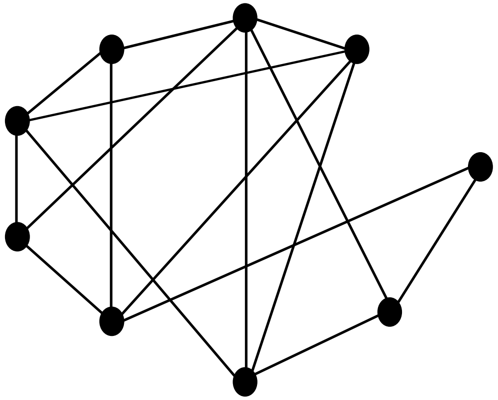
(i)

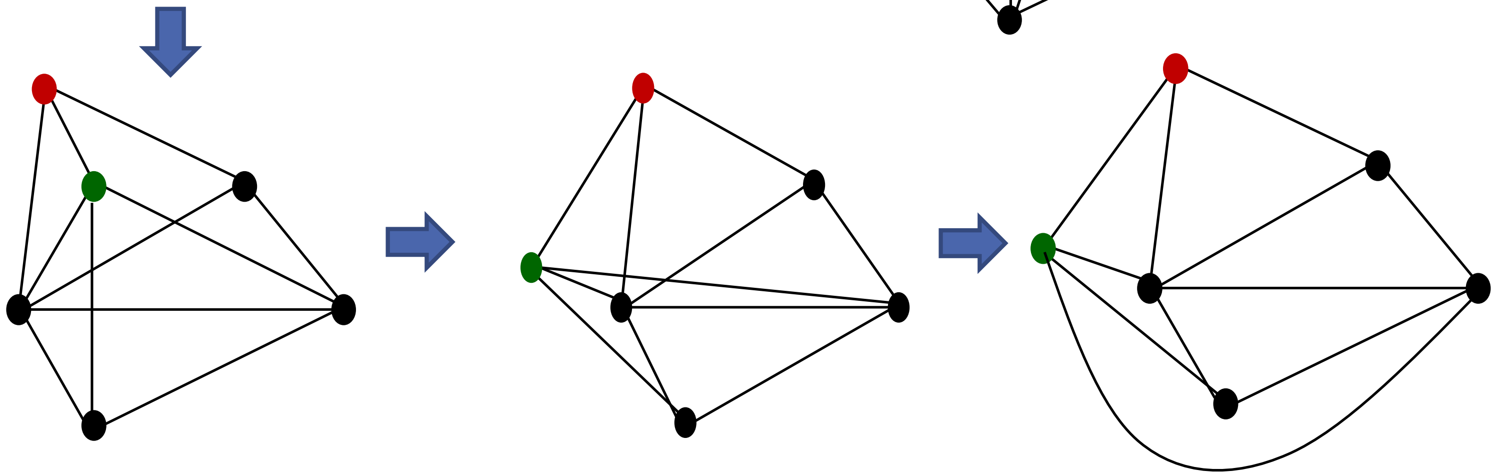


(ii)



(iii)





A. Only (i).

B. Only (ii).

C. Only (i) and (ii).

D. Only (i) and (iii).

E. None of the options (A),(B),(C),(D) is correct.

2nd Hardest MCQ

13. Let $f : A \to A$ be a bijection. Which of the following are true?

(i) If the order of $f \circ f$ is 1, then the order of f is 2.
(ii) If the order of $f \circ f$ is 2, then the order of f is 4.
(iii) If the order of f is 2, then the order of $f \circ f$ is 1.
(iv) If the order of f is 4, then the order of $f \circ f$ is 2.

A. Only (i) and (ii).
B. Only (iii) and (iv).
C. Only (i), (ii) and (iii).
D. Only (ii), (iii) and (iv).
E. None of the options (A),(B),(C),(D) is correct.

(i) False. If $f = id_A$, then $f \circ f = f = id_A$.
(ii) True. Order of f cannot be 1 or 2, otherwise $f \circ f = id_A$. Order of f cannot be 3, otherwise $f \circ f \circ f \circ f = id_A \circ f = f \neq id_A$.
(iii) True. $f \circ f = id_A$ and $f \neq id_A$.
(iv) True. $f \circ f \circ f \circ f = id_A$ and $f \circ f \neq id_A$.

23. Denote S_n as a set of all positive integers from 1 up to n inclusive (i.e. $\{1,2,3,\dots,n\}$). A bijection $\sigma : S_n \to S_n$ is called a *permutation*. A popular way to express permutations is via *cycle notation*.

To write down the permutation $\sigma : S_n \to S_n$ in cycle notation, one proceeds as follows:

1. Write an opening parenthesis followed by an arbitrary element $x \in S_n$. Example: $(\ x$
2. Trace the "orbit" of x, i.e. write down the values under successive applications of σ. Example: $(\ x \quad \sigma(x) \quad \sigma(\sigma(x)) \quad \dots$
3. Repeat until the value returns to x, and close the parenthesis without writing x (again). Example: $\left(x \quad \sigma(x) \quad \sigma(\sigma(x)) \quad \sigma\left(\sigma(\sigma(x))\right)\right)$
4. Continue with an element $y \in S_n$ which was not yet written, and repeat the above process. Example: $\left(x \quad \sigma(x) \quad \sigma(\sigma(x)) \quad \sigma\left(\sigma(\sigma(x))\right)\right)(\ y \dots)$
5. Repeat until all elements of S_n are written out.

For example, for a permutation $\sigma : S_6 \to S_6$ such that

$$\left(\sigma(1),\sigma(2),\sigma(3),\sigma(4),\sigma(5),\sigma(6)\right) = (2,6,5,4,3,1),$$

its cycle notation would be $(\mathbf{1\ 2\ 6})(\mathbf{3\ 5})(\mathbf{4})$.

23. (a) Write out the cycle notation for this permutation:

$$\sigma : S_4 \to S_4 \text{ such that } \big(\sigma(1), \sigma(2), \sigma(3), \sigma(4)\big) = (3,4,1,2).$$

$$\mathbf{(1\ 3)\ (2\ 4)}$$

(b) Write out the cycle notation for this permutation:

$$\sigma : S_6 \to S_6 \text{ such that } \big(\sigma(1), \sigma(2), \sigma(3), \sigma(4), \sigma(5), \sigma(6)\big) = (5,4,3,6,1,2).$$

$$\mathbf{(1\ 5)\ (2\ 4\ 6)\ (3)}$$

(c) Given two S_6 permutations σ_1 and σ_2 below, what is $\sigma_1 \circ \sigma_2$ in cycle notation?

$\sigma_1 = (1\ 2\ 6)(3)(4)(5)$ and $\sigma_2 = (1)(3\ 6)(2\ 4)(5)$.

$$\mathbf{(1\ 2\ 4\ 6\ 3)\ (5)}$$

Working:

$$\sigma_1 \circ \sigma_2(1) = \sigma_1\big(\sigma_2(1)\big) = \sigma_1(1) = 2$$
$$\sigma_1 \circ \sigma_2(2) = \sigma_1\big(\sigma_2(2)\big) = \sigma_1(4) = 4$$
$$\sigma_1 \circ \sigma_2(3) = \sigma_1\big(\sigma_2(3)\big) = \sigma_1(6) = 1$$
$$\sigma_1 \circ \sigma_2(4) = \sigma_1\big(\sigma_2(4)\big) = \sigma_1(2) = 6$$
$$\sigma_1 \circ \sigma_2(5) = \sigma_1\big(\sigma_2(5)\big) = \sigma_1(5) = 5$$
$$\sigma_1 \circ \sigma_2(6) = \sigma_1\big(\sigma_2(6)\big) = \sigma_1(3) = 3$$

24a. An edge weight matrix W of a simple undirected graph has each entry $w_{i,j}$ representing the weight of the edge between vertex i and vertex j. The edge weight matrix of a simple undirected graph is given below. What is the weight of its minimum spanning tree?

$$W = \begin{bmatrix} 0 & 3 & 8 & 3 & 5 \\ 3 & 0 & 4 & 3 & 5 \\ 8 & 4 & 0 & 4 & 6 \\ 3 & 3 & 4 & 0 & 5 \\ 5 & 5 & 6 & 5 & 0 \end{bmatrix}$$

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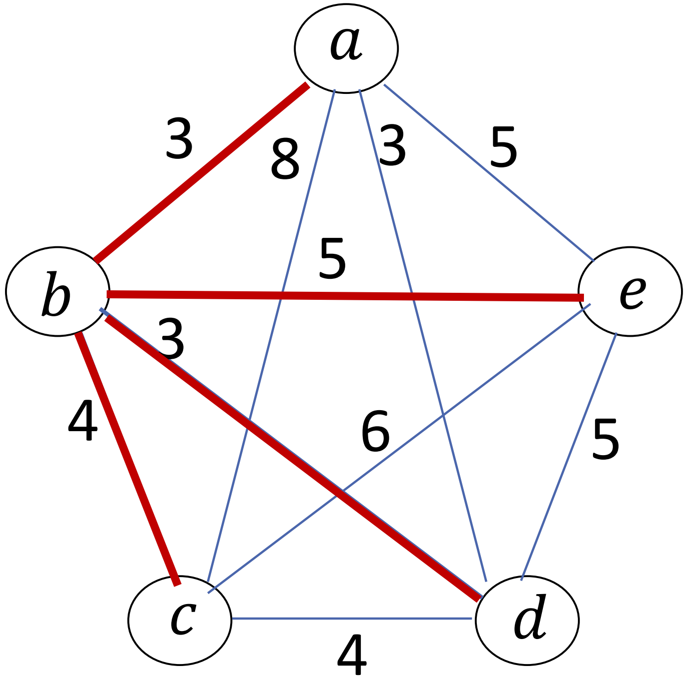

24b. The adjacency matrix A of a directed graph with the vertex set $V = \{v_1, v_2, v_3\}$ is given below, where the rows from top to bottom and columns from left to right are v_1, v_2, v_3 in that order. The entry $a_{i,j}$ represents the number of directed edges from v_i to v_j. Find the number of walks of length 4 from v_1 to v_3.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \qquad A^2 = \begin{bmatrix} 5 & 4 & 4 \\ 4 & 7 & 6 \\ 2 & 3 & 6 \end{bmatrix} \qquad A^4 = \begin{bmatrix} 49 & 60 & 68 \\ 60 & 83 & 94 \\ 34 & 47 & 62 \end{bmatrix}$$

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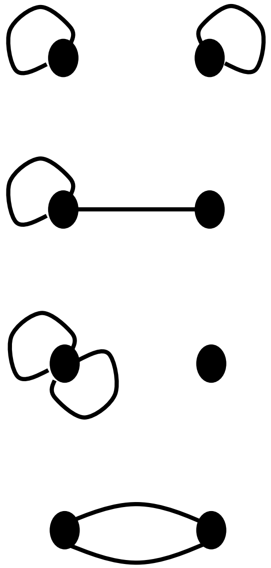
24c. State whether the following statements are true or false.

(i) Graph G has an Euler circuit. If Aiken removes an edge from G, the new graph G' is connected and has an Euler trail.

True

(ii) There are four non-isomorphic undirected graphs that have two vertices and two edges.

True



(iii) All simple undirected graphs with n vertices, where $n > 3$, can have degree $n - 2$ for every vertex.

False.

(If n is odd, total degree = $n(n-2)$ is odd.)

24d. A connected simple undirected graph G has n number of vertices (here n is even). Aiken removes one edge from graph G, and the new graph G' has 2 connected components, both of which are complete graphs. Find the minimum number of edges in G.

$$\frac{n(n-2)}{4} + 1$$

Suppose the two connected components contain n_1 and $n - n_1$ number of vertices respectively. Here $n_1 \in \mathbb{Z}^+$.

Number of edges in $G' = N(E_{G'}) = \frac{n_1(n_1-1)}{2} + \frac{(n-n_1)(n-n_1-1)}{2}$ as both components are complete graphs K_{n_1}, K_{n-n_1} (given in the question).

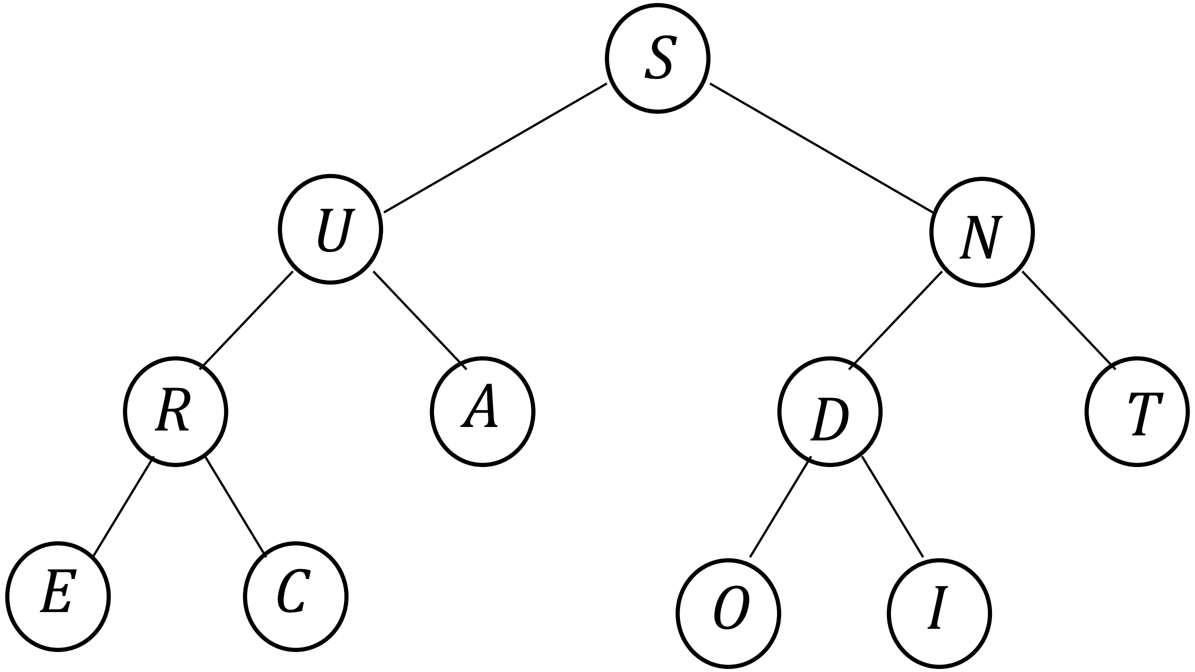
Simplify $N(E_{G'}) = \frac{(n_1^2-n_1)+(n-n_1)^2-(n-n_1)}{2} = \frac{2n_1^2+n^2-2nn_1-n}{2}$.

$\frac{dE}{dn_1} = \frac{4n_1-2n}{2} = 0$. For minimum, $4n_1 - 2n = 0 \rightarrow n_1 = \frac{n}{2}$.

Hence minimum number of edges in G is $N(E_G) = 2 \times \frac{\left(\frac{n}{2}\right)\left(\frac{n}{2}-1\right)}{2} + 1 = \frac{n(n-2)}{4} + 1$.

24e. The pre-order traversal and post-order traversal of a full binary tree are given below. What is the in-order traversal of the tree?

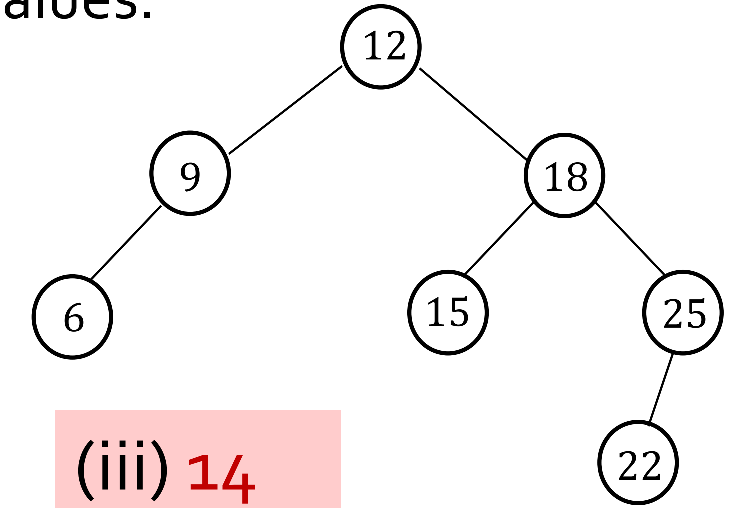
Pre-order: S U R E C A N D O I T
Post-order: E C R A U O I D T N S



In-order: **E R C U A S O D I N T**

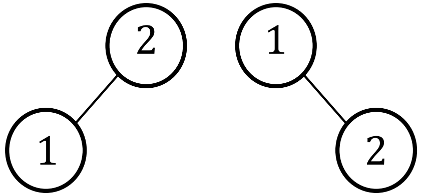
24f. A *Binary Search Tree* (BST) is a specialized form of a binary tree characterized by the following property: for any given vertex with the value x, all elements in its left subtree are smaller than x, and all elements in its right subtree are greater than x. You may assume that there are no duplicate values.

The diagram on the right shows an example of a BST with 7 values.

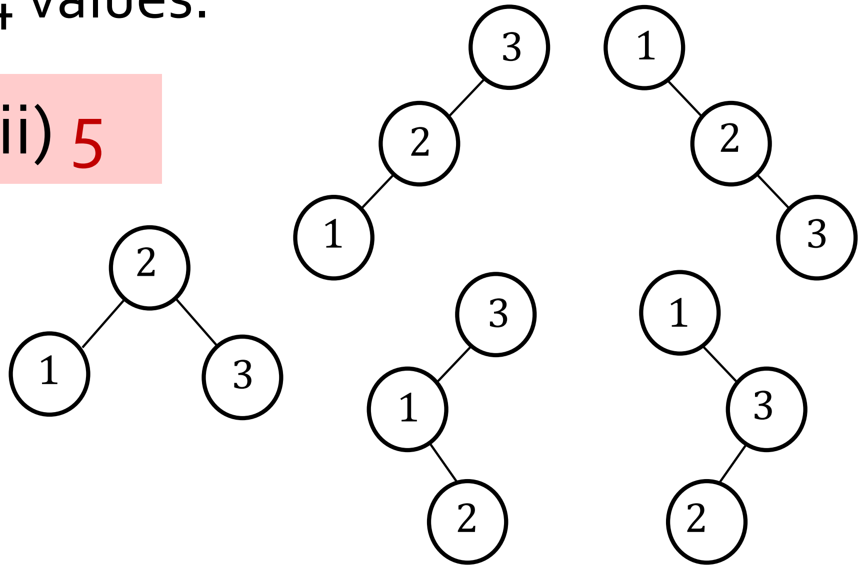


Find the number of distinct BSTs with (i) 2 values, (ii) 3 values, (iii) 4 values.

(i) 2



(ii) 5



(iii) 14

Catalan numbers

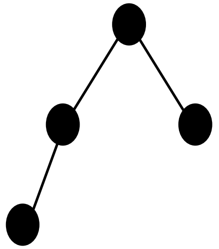
$$C_n = \frac{(2n)!}{(n+1)!\,n!}$$

1,2,5,14,42,132,429,1430,4862, …

24g. State whether the following statements are true or false.

(i) A binary tree has k terminal vertices (leaves) and $k+1$ internal vertices.

False



(ii) If all the edge weights are distinct in a graph, Prim's and Kruskal's algorithms produce the same minimum spanning tree.

True

(iii) An edge e is contained in every spanning tree for a connected graph G if and only if removal of e disconnects G.

True

25a. A loaded coin has the probability of 0.6 for getting a head. It is tossed five times. What is the probability of getting exactly three heads? Write your answer as a single number with four significant digits.

$$\binom{5}{3}(0.6)^3(0.4)^2 = 10 \times 0.216 \times 0.16 = \mathbf{0.3456.}$$

25b. A fair coin is tossed n times, where $n > 10$. Let $1/k$ denote the probability of getting an odd number of heads. What is k? Write your answer as a single number.

$k = \mathbf{2}$.

Note that $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \binom{n}{7} + \cdots = 2^n/2 = 2^{n-1}$.

25c. Morse code uses dashes and dots to encode the alphanumeric characters. How many distinct messages can be represented using five dashes and three dots? Write your answer as a single number.

$$8!/(5!\,3!) = \mathbf{56}$$

25d. What is the total number of ways in which 16 identical tasks can be allotted to 4 distinct processors such that every processor is allotted an even number of tasks? Write your answer as a single number.

$$\binom{8+4-1}{4-1} = \binom{11}{3} = \mathbf{165}$$

This is same as distributing 8 tasks (we can put two tasks together to make it even) among 4 processors. Thus, we want to find total number of solutions for the equation $w + x + y + z = 8$.

25e. An e-commerce platform uses either Apple ID or Google account as a third-party authenticator to register users on the platform. Currently, 85% of the user's log-in use the Google credentials whereas the rest of them use their Apple ID. A welcome bonus of $10 is sent to 95% of the Apple ID registered users. The same bonus is sent to 80% of the Google users. What is the probability that a user has used Google account for authentication given that he or she has received the welcome bonus? Write your final answer as a single number with four significant digits.

$P(Google) = 0.85$

$P(Apple) = 0.15$

$P(Bonus|Google) = 0.8$

$P(Bonus|Apple) = 0.95$

$$P(Bonus) = P(Bonus|Google) \cdot P(Google) + P(Bonus|Apple) \cdot P(Apple)$$
$$= (0.8 \times 0.85) + (0.95 \times 0.15) = 0.8225$$

$$P(Google|Bonus) = \frac{P(Bonus|Google) \cdot P(Google)}{P(Bonus)} = \frac{0.8 \times 0.85}{0.8225} = \mathbf{0.8267}$$

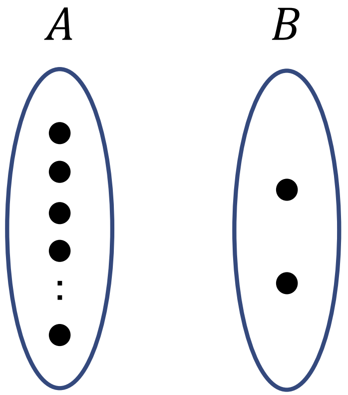
25f. You have written a five-digit cheque number on a blank paper and passed the sheet to the cashier in the bank. Note that the number may start with a 0, for example, 00123.

The cashier is unable to tell the number due to the lack of orientation of the blank sheet. How many *distinct* five-digit numbers can be written that can be read both ways: right side up or upside down (i.e. when rotated 180 degrees)? For instance: 09168 can be read as 89160 and vice versa**.** The 5 digits that can be read both ways are 0,1,6,8,9. Write your answer as <u>a single number</u>.

Tutorial 9 Q2.

25g. How many surjective functions are there from a non-empty finite set A to set B under each of the following conditions.

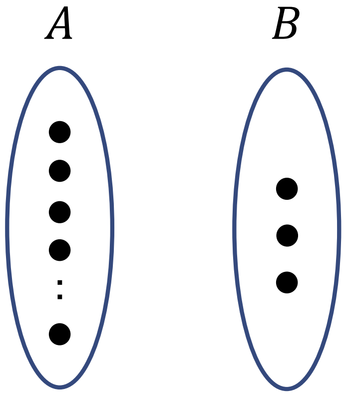
(i) $|A| = n$ and $|B| = 2$.



$\mathbf{2^n - 2}$

Explanation. 2^n functions are possible, of which two are not surjective (all elements in A map to either of the elements in B).

(ii) $|A| = n \geq 2$ and $|B| = 3$.



$\mathbf{3^n - (3 \cdot 2^n) + 3}$

Explanation. Let $B = \{x, y, z\}$. 3^n functions are possible, of which 2^n do not map to x, 2^n do not map to y, 2^n do not map to z. Hence, $3 \cdot 2^n$ are not surjective. But we have double counted 3 functions that map all n elements to $x, y, or\ z$.

26. Consider the Fibonacci function:

$$F(0) = 0; \qquad F(1) = 1; \qquad F(n+1) = F(n) + F(n-1), n \geq 1.$$

One interesting property of this function can be expressed as follows:

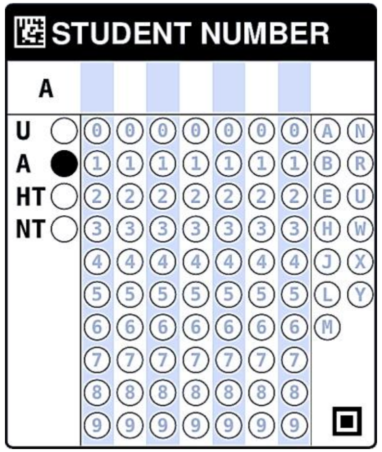
$$P(a,b) \equiv F(a+b) = \big(F(a+1) \times F(b) + F(a) \times F(b-1)\big),\ \forall a \geq 0, b \geq 1.$$

Tutorial 7 Q8.

AY2024/25 Semester 1 Exam Paper

Date: 29 November 2024, Friday, 2:30 – 4:30pm

Venue: MPSH2



Important! Shade your Student Number CORRECTLY with a pencil (2B or above).

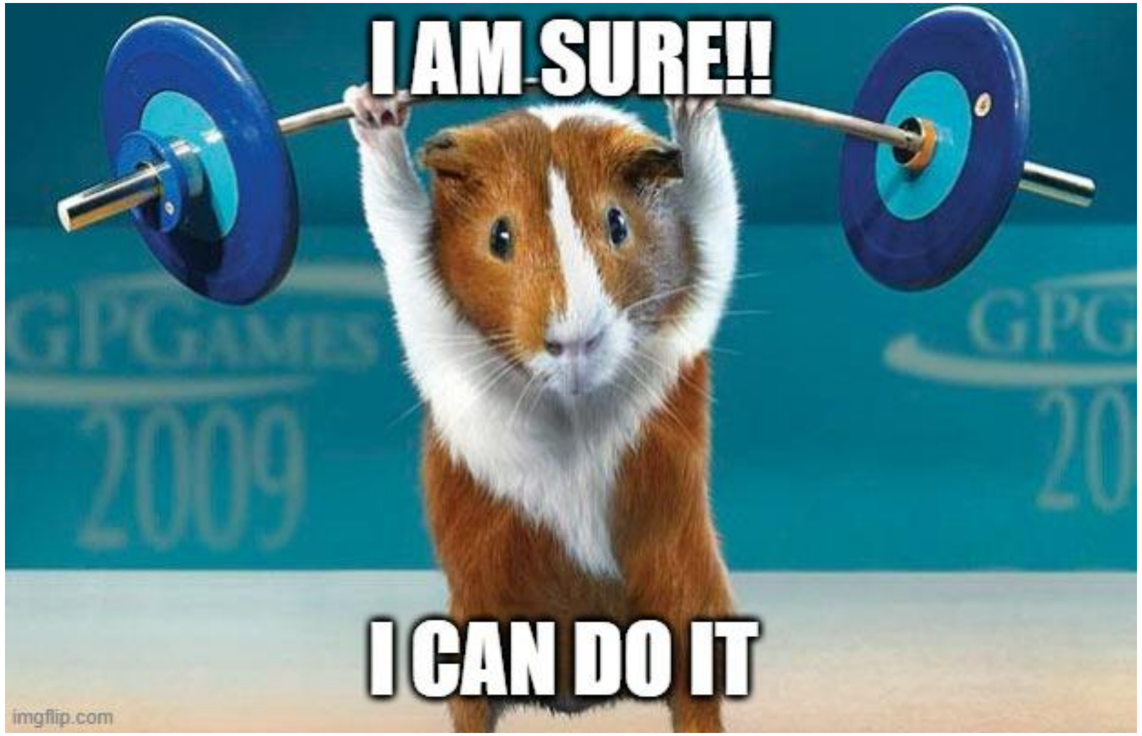
Format:

- 23 MCQs: 46 marks
- Q24: 20 marks
- Q25: 20 marks
- Q26: 14 marks

All topics tested. 30% on older topics (tested in midterm), 70% on newer topics.

Things to bring:

- Student Card
- Pencil (2B or above), eraser, writing papers
- Calculators (NUS approved)


I AM SURE!!
I CAN DO IT
imgflip.com


GOOD LUCK ON THE EXAM
HOPE IT ISN'T TOO RUFF!

THE END