CS1231S Discrete Structures

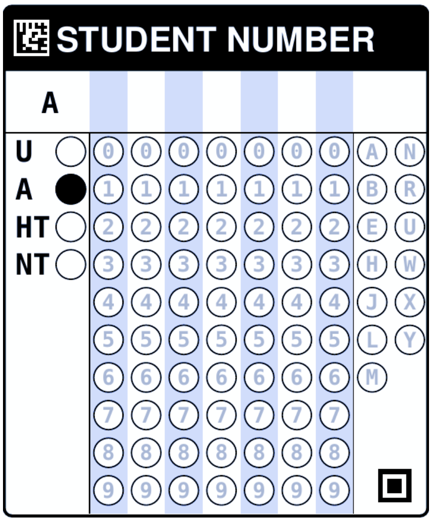
Midterm Test — Answer Sheet

AY2022/23 Semester 2

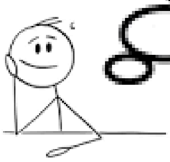
Time allowed: 1 hour 30 minutes

INSTRUCTIONS

1. Write your **Student Number** on the right AND, using pen or pencil, shade the corresponding circle **completely** in the grid for each digit or letter. DO NOT WRITE YOUR NAME!
2. Zero mark will be given if you write/shade your Student Number incompletely or incorrectly.
3. Write your Student Number at the top of page 3.
4. This answer sheet comprises **FOUR (4) pages**.
5. All questions must be answered in the space provided; no extra sheets will be accepted as answers.
6. You must submit only this **ANSWER SHEET** and no other documents.
7. An excerpt of the question may be provided to aid you in answering in the correct box. It is not the exact question. You should still refer to the original question in the question paper.
8. You may write your answers using pencil (at least 2B) or pen as long as it is legible (no red ink, please).
9. The maximum mark for this paper is 50.
10. **Marks may be deducted** for (i) illegible handwriting, and/or (ii) excessively long answer.
11. Each multiple choice question is intended to have only one answer. Please shade the appropriate bubble.



For Examiner's Use Only		
Question	**Marks**	**Remarks**
Q1-12	/ 24	
Q13	/ 5	
Q14	/ 6	
Q15	/ 3	
Q16	/ 5	
Q17	/ 7	
Total	**/ 50**	



Any feeling about this midterm test or CS1231S? This will not be graded! 😊

Part A: Multiple Choice Questions (Total: 24 marks)

Please shade using **pencil** only ONE bubble for each question.

	(A)	(B)	(C)	(D)	(E)
1.	○	●	○	○	○
2.	●	○	○	○	○
3.	○	○	●	○	○
4.	○	○	○	●	○
5.	○	●	○	○	○
6.	○	○	○	○	●
7.	○	○	○	○	●
8.	○	○	○	○	●
9.	○	●	○	○	○
10.	○	○	●	○	○
11.	○	○	○	○	●
12.	○	○	●	○	○
	(A)	(B)	(C)	(D)	(E)

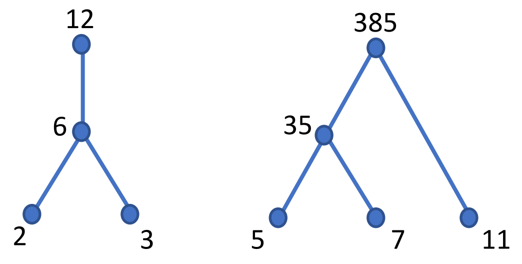
Part B (Total: 26 marks)

Maximal chain has length of 4.
Many possible answers. Two are given below.

13. Maximal chains. [5 marks]

(a) $\{\emptyset, \{a\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}\}$ | $\{\emptyset, \{b\}, \{b,d\}, \{b,c,d\}, \{a,b,c,d\}\}$ [2]

(b) Hasse diagram: [3]



Two maximal chains of different lengths:

{2,6,12} and {11,385}

or {3,6,12} and {11,385}
or {5,35,385} and {11,385}
or {7,35,385} and {11,385}

14. Propositional logic. [6 marks]

1.	$(q \vee p)$	(Commutative law on P1)
2.	$(\sim q \vee r)$	(Implication law on P2)
3.	$(\sim(p \wedge s) \vee t)$	(Implication law on P3)
4.	$\sim r$	(P4)
5.	$\sim(\sim q) \vee (u \wedge s)$	(Implication law on P5)
6.	$\big(q \vee (u \wedge s)\big)$	(Double negative law on (5))
7.	$\sim t$	(Negation of conclusion) ← Start of proof by contradiction
8.	$(p \vee r)$	(Resolution law on (1) and (2))
9.	p	(Elimination on (4) and (8))
10.	$(\sim p \vee \sim s \vee t)$	(De Morgan's law on (3))
11.	$(\sim s \vee t)$	(Elimination on (9) and (10))
12.	$\big((q \vee u) \wedge (q \vee s)\big)$	(Distributive Law on (6))
13.	$(q \vee s)$	(Specialization on (12))
14.	$(s \vee r)$	(Resolution law on (2) and (13))
15.	$(r \vee t)$	(Resolution law on (11) and (14))
16.	r	(Elimination on (7) and (15))
17.	$false$	(Negation law on (4) and (16)) ← Contradiction arrived

(Note: Elimination rule of inference (Lecture 2 slide 64): $(p \vee q) \wedge \sim q \rightarrow p$.)

15. Sets. [3 marks]

(a) $A \cap B = \emptyset$. Example: $A = \{a\}, B = \{b\}$.
Then $|A \cup B| = |\{a, b\}| = 2 = 1 + 1 = |A| + |B|$. [1]

(b) $A \cap B \neq \emptyset$. Example: $A = \{a, c\}, B = \{b, c\}$.
Then $|A \cup B| = |\{a, b, c\}| = 3 \neq 4 = 2 + 2 = |A| + |B|$. [1]

(c) **False** [1]

16. Relations. [5 marks]
Write capital **T** (for true) or capital **F** (for false) clearly in the cells below.

	Reflexive	Irreflexive	Symmetric	Antisymmetric	Asymmetric	Transitive
R_1	T	F	T	F	F	T
R_2	F	T	T	F	F	F
R_3	F	F	T	F	F	F
R_4	F	F	F	T	F	T
R_5	F	T	T	T	T	T

17. Relations [7 marks]

(a) [3]

To show that R is an equivalence relation, it suffices to show that R is reflexive if for every $x \in A$, there exists $y \in A$ such that xRy.

1. Let $x \in A$.
2. There exists $y \in A$ such that xRy (as given).
3. Since xRy, we have yRx (by symmetry, as given).
4. Since we have xRy and yRx, we have xRx (by transitivity, as given).
5. Hence R is reflexive (by the definition of reflexivity).
6. Therefore, R is an equivalence relation (as it is reflexive, symmetric and transitive).

(b) (i) [2]

Yes, S is an equivalence relation.
$|B/S| = 9$. (Each equivalence class is a singleton.)

(ii) [2]

Yes, S is a partial order. The maximal and minimum elements are the same: 2, 3, 5, 7, 11, 13, 17, 19, 23.

=== END OF PAPER ===