CS1231S Discrete Structures

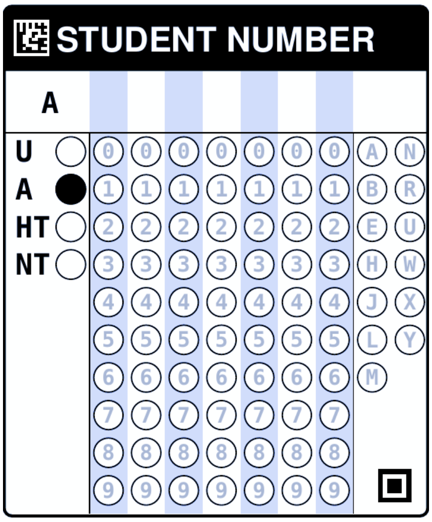
Midterm Test — Answer Sheet

AY2023/24 Semester 1

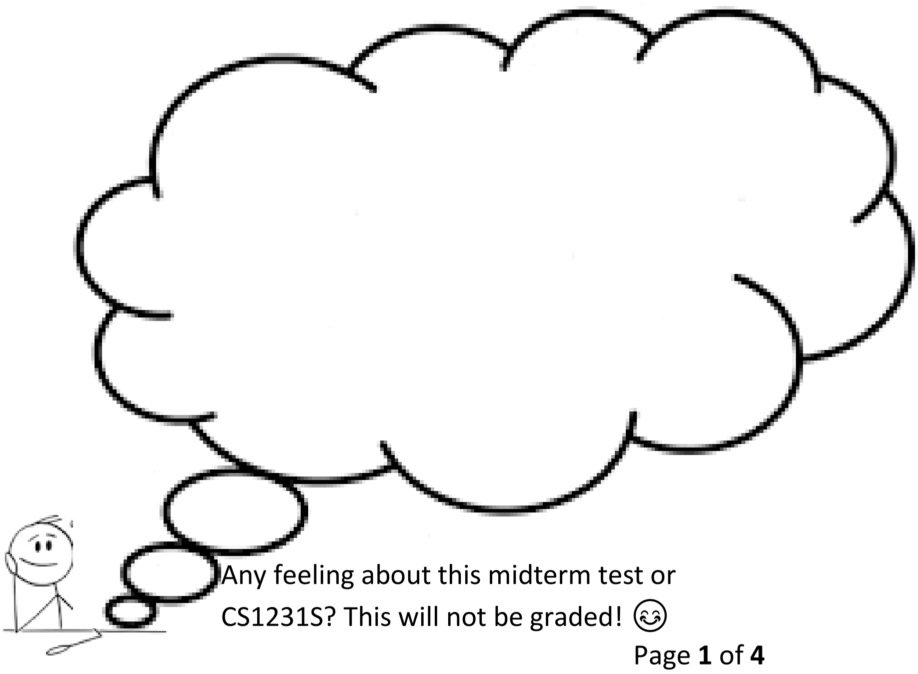
Time allowed: 1 hour 30 minutes

INSTRUCTIONS

1. Write your **Student Number** on the right AND, using pen or pencil, shade the corresponding circle **completely** in the grid for each digit or letter. DO NOT WRITE YOUR NAME!
2. Zero mark will be given if you write/shade your Student Number incompletely or incorrectly.
3. Write your Student Number at the top of page 3.
4. This answer sheet comprises **FOUR (4) pages**.
5. All questions must be answered in the space provided; no extra sheets will be accepted as answers.
6. You must submit only this **ANSWER SHEET** and no other documents.
7. An excerpt of the question may be provided to aid you in answering in the correct box. It is not the exact question. You should still refer to the original question in the question paper.
8. You may write your answers using pencil (at least 2B) or pen as long as it is legible (no red ink, please).
9. The maximum mark for this paper is 50.
10. **Marks may be deducted** for (i) illegible handwriting, and/or (ii) excessively long answer.
11. Each multiple choice question is intended to have only one answer. Please shade the appropriate bubble.



For Examiner's Use Only		
Question	**Marks**	**Remarks**
Q1-15	/ 30	
Q16	/ 6	
Q17	/ 4	
Q18	/ 4	
Q19	/ 6	
Total	**/ 50**	



Any feeling about this midterm test or CS1231S? This will not be graded! 😉

This page is intentionally left blank

Explanation

2. Only (i) and (ii) are true.

3. (i) $\big(p \vee (p \rightarrow {\sim}q)\big)$ is a tautology but ${\sim}(p \rightarrow q)$ is not. Let $p \equiv q \equiv true$, then ${\sim}(p \rightarrow q) \equiv false$.
   (iii) Let $p \equiv q \equiv r \equiv true$ and $s \equiv false$, then $\big((p \leftrightarrow q) \wedge (q \rightarrow r) \wedge (r \vee {\sim}s) \wedge ({\sim}s \rightarrow q)\big) \rightarrow s \equiv false$.

4. Aiken and Dueet are knaves. Noasu is a knight.

5. (i) False. Counterexample: $x = 0$.
   (ii) False. Case: $x \geq 0$, let $y = x + 1$; case: $x < 0$, let $y = x - 1$. In both cases, $x^2 \not\geq y^2$.
   (iii) True. Eg: Let $x = 0$, for any y let $z = 0$. Or, let $x = 1$, for any y let $z = y$. (In essence, $z = xy$).

6. $\emptyset \cup \mathcal{P}(\emptyset) = \mathcal{P}(\emptyset) = \{\emptyset\}$.
   (i) $\emptyset \times \{\emptyset\} = \emptyset$.
   (ii) $\mathcal{P}\big(\emptyset \cup \mathcal{P}(\emptyset)\big) = \mathcal{P}\big(\mathcal{P}(\emptyset)\big) = \mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$.
   (iii) $\mathcal{P}(\emptyset) \setminus \emptyset = \mathcal{P}(\emptyset) = \{\emptyset\}$.
   (iv) $(\emptyset \cap \{\emptyset\}) \cup (\emptyset \cup \{\emptyset\}) = \emptyset \cup \{\emptyset\} = \{\emptyset\}$.

7. (iii) Let $A = B$. Then $A \cup B \ = \ A \cap B$.
   (iv) Let $A \cap B = \emptyset$. Then $A \oplus B = (A \setminus B) \cup (B \setminus A) = (A \cap \bar{B}) \cup (B \cap \bar{A}) = A \cup B$.

8. (ii) Counterexample: $B = \{A\}, C = \{A, X\}$ but $A \not\subseteq C$.
   (iii) Counterexample: $A = \{x\}, B = \{x, y\}, C = \{\,\{x, y\}\,\}$ but $A \notin C$.
   (iv) Counterexample: $A = \{x\}, B = \{x, y\}, C = \{\,\{x, y\}\,\}$ but $A \not\subseteq C$.

9. Counterexample for (ii), (iii), (iv): $A = \{a, b, c\}, R = \{(a, a), (b, b), (c, c), (a, c)\}$ and $S = \{(a, b), (b, a)\}$.
   (ii) $R \circ S = \{(a, b), (b, a), (b, c)\}$ is not symmetric.
   (iii) $R \cup S = \{(a, a), (b, b)(c, c), (a, c), (a, b), (b, a)\}$ is not symmetric.
   (iv) $(R \circ S)^{-1} = \{(b, a), (a, b), (c, b)\}$ is not transitive.

10. (i) True. Tutorial 5 Q6.
    (ii) False. Let $A = \{a, b\}$ and $R = \{(a, b)\}$. Then R is not reflexive but it is asymmetric.
    (iii) False. Let $A = \{a, b\}$ and $R = \{(a, a)\}$. Then R is not reflexive but it is not asymmetric.
    (iv) False. The relation $=$ on $\mathbb{Z}$ is both an equivalence relation and a partial order.

11. $R \circ R = \{(1,2),\ (1,4),\ (3,3),\ (4,2),\ (4,4)\}$. $R \circ R \circ R \circ R = (R \circ R) \circ (R \circ R) = \{(1,2),\ (1,4),\ (3,3),\ (4,2),\ (4,4)\}$.

12. (i) Always true, because equivalence relations are symmetric. R is symmetric if and only if $R = R^{-1}$ (Tutorial 4 Q2).
    (ii) Always true. Proof: $R \circ R^{-1} = R \circ R$ (since $R = R^{-1}$) $= R$ (by parts iii and iv).

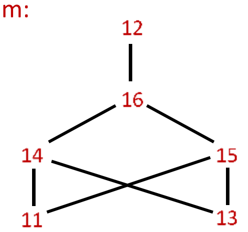
    (iii) Always true, because equivalence relations are transitive. Proof:
       1. Let $(x, z) \in R \circ R$.
       2. There is some y in the domain such that xRy and yRz (by definition of composition).
       3. So, xRz (by transitivity). That is, $(x, z) \in R$.

    (iv) Always true, because equivalence relations are reflexive. Proof:
       1. Let $(x, z) \in R$.
       2. Then, $(x, x) \in R$ (by reflexivity).
       3. So, when $y = x$, we have xRy and yRz.
       4. So, $(x, z) \in R \circ R$ (by definition of composition).

13. (i) The poset $(\{a\}, \{(a, a)\})$ has a as both smallest and largest.
    (ii) The poset $(\{a, b\}, \{(a, a), (b, b)\})$ has a and b that are both maximal and minimal but neither largest nor smallest.
    (iii) If two distinct maximal elements are comparable, then it violates the definition of maximality for one of them.
    (iv) The poset $(\mathbb{Z}, \leq)$ has no maximal or minimal elements.

14. P is not reflexive because $(3,3) \notin P$.
    R^t is not antisymmetric because by transitive closure, $(1,2), (2,3) \in R$ implies $(1,3) \in R^t$. However, $(3,1) \in R \subseteq R^t$ also.

15. Hasse diagram:



Student Number: **A** | | | | | | | |

Part A: Multiple Choice Questions (Total: 30 marks)

Please shade using **pencil** only ONE bubble for each question.

	(A)	(B)	(C)	(D)	(E)
1.	●	○	●	○	○
2.	○	○	○	○	●
3.	●	○	○	○	○
4.	○	●	○	○	○
5.	○	●	○	○	○
6.	○	○	○	●	○
7.	○	●	○	○	○
8.	●	○	○	○	○
9.	●	○	○	○	○
10.	●	○	○	○	○
11.	○	●	○	○	○
12.	○	○	○	●	○
13.	○	○	●	○	○
14.	○	●	○	○	○
15.	○	●	○	○	○
	(A)	(B)	(C)	(D)	(E)

Part B (Total: 20 marks)

16. [6 marks]

(a) [2] **1**

(b) [2] **8, 9**

(c) [2]

$$y = \frac{1}{|x|+1} \qquad y = -x - \frac{1}{2}$$

$$y = \begin{cases} 1 & if\ x = 0; \\ -x & if\ x \neq 0. \end{cases} \qquad y = \begin{cases} \dfrac{1}{x+1} & if\ x > 0; \\ 1 & if\ x \leq 0. \end{cases}$$

Many other possible answers.

17. [4 marks]

(a) [1] $(A_1 \cap A_2) \setminus A_3$

$\emptyset$

(b) [1] $(A_1 \cup A_2) \setminus A_3$

$\{ \emptyset \}$

(c) [1] $(A_1 \times A_2) \cup A_3$

$\{ (\emptyset, a), a, \{a\} \}$

(d) [1] $(A_1 \cup A_2) \times A_3$

$\{ (\emptyset, a), (\emptyset, \{a\}), (a, a), (a, \{a\}) \}$

18. [4 marks]

(a) [1] Minimal: **11, 13**

(b) [1] Smallest: **None**

(c) [1] Maximal: **12**

(d) [1] Largest: **12**

19. [6 marks]

(a) [3]

1. Let $a, b, c \in \mathbb{Q}$ such that aSb and bSc.
2. There exists some $m, n \in \mathbb{Z}$ such that $a - b = \frac{1}{2}m$ and $b - c = \frac{1}{2}n$ (by definition of S).
3. Now, $a - c = (a - b) + (b - c) = \frac{1}{2}m + \frac{1}{2}n = \frac{1}{2}(m + n)$ (by basic algebra).
4. $m + n \in \mathbb{Z}$ (by closure of integers under addition).
5. Therefore, aSc (by definition of S).

(b) [1]

[0] and [0.1] (Many possible answers)

(c) [2]

$$\{\, [x] : 0 \le x < \frac{1}{2} \land x \in \mathbb{Q} \,\}$$

=== END OF PAPER ===