CS1231S Discrete Structures

Midterm Test — Answer Sheets

AY2024/25 Semester 1

Time allowed: 1 hour 30 minutes

INSTRUCTIONS

1. Write your **Student Number** on the right AND, using a **pencil** (2B or above), shade the corresponding circle **completely** in the grid for each digit or letter. DO NOT WRITE YOUR NAME!
2. Zero mark will be given if you write/shade your Student Number incompletely or incorrectly.
3. Write your Student Number at the top of page 3.
4. There are **FOUR (4) pages** in the Answer Sheets.
5. All questions must be answered in the space provided; no extra sheets will be accepted as answers.
6. You must submit only these **ANSWER SHEETS** and no other documents.
7. An excerpt of the question may be provided to aid you in answering in the correct box. It is not the exact question. You should still refer to the original question in the question paper.
8. You may write your answers in pencil (2B or above) or pen. Pencil is preferred in case you need to erase and rewrite your answers.
9. The maximum mark for this paper is 50.
10. **Marks may be deducted** for (i) illegible handwriting, and/or (ii) excessively long answer.
11. Each multiple choice question is intended to have only one answer. Please shade the appropriate bubble in **pencil**.

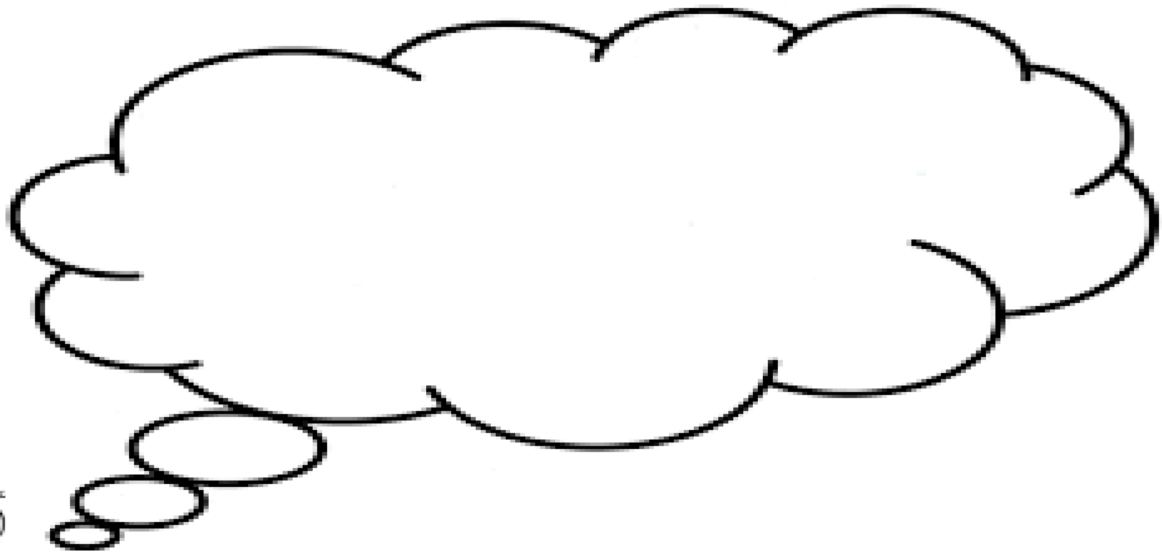
STUDENT NUMBER

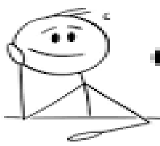
A

U ○	0	0	0	0	0	0	0	A	N
A ●	1	1	1	1	1	1	1	B	R
HT ○	2	2	2	2	2	2	2	E	U
NT ○	3	3	3	3	3	3	3	H	W
	4	4	4	4	4	4	4	J	X
	5	5	5	5	5	5	5	L	Y
	6	6	6	6	6	6	6	M	
	7	7	7	7	7	7	7		
	8	8	8	8	8	8	8		
	9	9	9	9	9	9	9		

For Examiner's Use Only

Question	Marks	Remarks
Q1-15	/ 30	
Q16	/ 5	
Q17	/ 15	
Total	**/ 50**	





Any feeling about this midterm test?

This will not be graded! 😉

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This page is for your rough work. Whatever is written on this page will NOT be graded.

Explanation for MCQs

Q3: (i), (ii) and (iii) are all true.

Q4. $\big((\cdots)\vee p\big)\wedge\big(p\rightarrow(p\rightarrow q)\big)\wedge p$

$\equiv\big((\cdots)\vee p\big)\wedge\Big(\big(p\rightarrow(p\rightarrow q)\big)\wedge p\Big)$	by associative law
$\equiv\big((\cdots)\vee p\big)\wedge\Big(p\wedge\big(p\rightarrow(p\rightarrow q)\big)\Big)$	by commutative law
$\equiv\Big(\big((\cdots)\vee p\big)\wedge p\Big)\wedge\big(p\rightarrow(p\rightarrow q)\big)$	by associative law
$\equiv\Big(p\wedge\big(p\vee(\cdots)\big)\Big)\wedge\big(p\rightarrow(p\rightarrow q)\big)$	by commutative law x2
$\equiv p\wedge\big(p\rightarrow(p\rightarrow q)\big)$	by absorption law
$\equiv p\wedge\big({\sim}p\vee({\sim}p\vee q)\big)$	by implication law x2
$\equiv p\wedge\big(({\sim}p\vee{\sim}p)\vee q\big)$	by associative law
$\equiv p\wedge({\sim}p\vee q)$	by idempotent law
$\equiv p\wedge q$	by variant absorption law (assignment 1)

Q5. $p\equiv r\equiv false$. (i), (ii) and (iii) are true. (iv) is false.

Q6. If $B=\emptyset$, then statement (iii) may not be true.

Q7. Note that $\{\{2,3\},\{4\},\{3,2\},\{1\}\}=\{\{2,3\},\{4\},\{1\}\}$.

Q8. Counter-example for (ii): $S=\{a,\{a\}\}$. Then $S\cap\mathcal{P}(S)=\{\{a\}\}$.

Q9. A relation can be both symmetric and antisymmetric. If it is also reflexive and transitive, then it is both an equivalence relation and a partial order. The above is one. Another example is the = relation on $\mathbb{Z}$.

Q10. $R=\{(true,true),(false,false),(false,true)\}$. R is reflexive, is not symmetric, is antisymmetric, and is transitive.

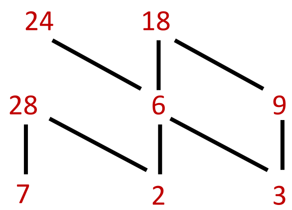
Q12. None of (i), (ii), (iii) are true. Let $A=\{a,b\}$.

(i) Counterexample: $R=\{(a,a),(a,b),(b,a)\}$. Then $|R\circ R|=|\{(a,a),(a,b),(b,a),(b,b)\}|=4$.

(ii) Counterexample: $R=\{(a,a),(a,b)\}$. Then $|R\circ R|=|\{(a,a),(a,b)\}|=2$.

(iii) Counterexample: $R=\{(a,a)\}$. Then $|R\circ R|=|\{(a,a)\}|=1$.

Q13. The longest maximal chains have length of 2 and they are {3,9,18}, {3,6,18}, {3,6,24}, {2,6,18}, {2,6,24}



Q14. (i) {2,3,28} is not an antichain as 2 and 28 are comparable.
(iii) {6} is an antichain (vacuously true).

Q15. (iii) is not a linearization as $9\preccurlyeq 18$.

Student Number: **A** | | | | | | | |

Part A: Multiple Choice Questions (Total: 30 marks)

Please shade only ONE bubble for each question using **pencil**.

	(A)	(B)	(C)	(D)	(E)
1.	○	●	○	○	○
2.	○	○	●	○	○
3.	○	○	○	○	●
4.	○	○	●	○	○
5.	○	○	●	○	○
6.	●	○	○	○	○
7.	○	○	●	○	○
8.	○	○	○	●	○
9.	○	○	●	○	○
10.	○	○	○	●	○
11.	○	●	○	○	○
12.	○	○	○	○	●
13.	○	●	○	○	○
14.	○	○	○	●	○
15.	○	●	○	○	○
	(A)	(B)	(C)	(D)	(E)

Part B (Total: 20 marks)

16. [5 marks]

(a) $\mathcal{P}(A) \setminus \mathcal{P}(\mathcal{P}(\emptyset)) = \{$ $\{x\}, \{\emptyset, x\}$ $\}$ [1]

(b) $\mathcal{P}\left((\mathcal{P}(A) \setminus \mathcal{P}(\emptyset)) \setminus \{A\}\right) = \{$ $\emptyset,\ \{\{\emptyset\}\},\ \{\{x\}\},\ \{\{\emptyset\}, \{x\}\}$ $\}$ [2]

(c) $(\mathcal{P}(\emptyset) \times \mathcal{P}(A)) \setminus (\mathcal{P}(A) \times \mathcal{P}(\emptyset)) = \{$ $(\emptyset, \{\emptyset\}),\ (\emptyset, \{x\}),\ (\emptyset, \{\emptyset, x\})$ $\}$ [2]

17. [15 marks]

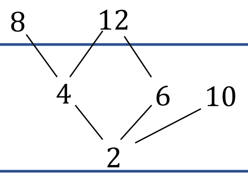
(a) [3] $|R^r| =$ 5 $|R^s| =$ 4 $|R^t| =$ 2

(b) [2] $T = \{$ $(\boldsymbol{a}, \boldsymbol{a})$, $(\boldsymbol{a}, \boldsymbol{d})$, $(\boldsymbol{a}, \boldsymbol{c})$, $(\boldsymbol{b}, \boldsymbol{c})$ $\}$

(c) [2]

Minimal:	Maximal:	Smallest:	Largest:
2	8,10,12	2	None

(d) [2] Maximal chains: $\{\mathbf{2}, \mathbf{4}, \mathbf{8}\}$ and $\{\mathbf{2}, \mathbf{10}\}$ or $\{\mathbf{2}, \mathbf{4}, \mathbf{12}\}$ and $\{\mathbf{2}, \mathbf{10}\}$ or $\{\mathbf{2}, \mathbf{6}, \mathbf{12}\}$ and $\{\mathbf{2}, \mathbf{10}\}$



(e) [2] $x(R_1 \circ R_2)y \;\Leftrightarrow\; x = \dfrac{\boldsymbol{y^2}}{\mathbf{4}}$

(f) [2] "$\forall x, y \in \mathbb{Z} \times \mathbb{Z}$, x and y are comparable" is False.

Counterexample: (1,3) and (2,4) are noncomparable, as neither $(1 \leq 2) \wedge (3 \geq 4)$ nor $(2 \leq 1) \wedge (4 \geq 3)$.

(g) [2] "$\forall x, y \in \mathbb{Z} \times \mathbb{Z}$, x and y are compatible" is True.

1. Let x be arbitrary $(x_1, x_2) \in \mathbb{Z} \times \mathbb{Z}$, and y be arbitrary $(y_1, y_2) \in \mathbb{Z} \times \mathbb{Z}$.
2. Let $c = (c_1, c_2) = (\max(x_1, y_1), \min(x_2, y_2))$
3. Then $(x_1 \leq c_1) \wedge (y_1 \leq c_1)$ (by definition of max) and $(x_2 \geq c_2) \wedge (y_2 \geq c_2)$ (by definition of min)
4. Then $(x_1, x_2)\ W\ c$ and $(y_1, y_2)\ W\ c$ (by definition of W)
5. Hence, $\forall x, y \in \mathbb{Z} \times \mathbb{Z}$, x and y are compatible.

=== END OF PAPER ===