NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

MID-TERM TEST
AY2024/25 Semester 1

CS1231S — DISCRETE STRUCTURES

9 October 2024 Time Allowed: 1 hour 30 minutes

INSTRUCTIONS

1. This assessment paper contains **SEVENTEEN (17)** questions in **TWO (2)** parts and comprises **SEVEN (7)** printed pages.
2. This is an **OPEN BOOK** assessment.
3. Printed/written materials are allowed. Apart from calculators, electronic devices are not allowed.
4. Answer **ALL** questions on the **ANSWER SHEETS** provided.
5. Shade your Student Number and MCQ answers **ONLY with a pencil** (2B or above).
6. You may write your answers in pen or pencil, though we advise you to use pencil in case you need to erase and rewrite your answers. Write legibly, or marks may be deducted.
7. You are to write within the space provided. No extra pages should be submitted as it will jam the scanning machine.
8. The maximum mark of this assessment is 50.
9. Do not start writing or flip over this page until you are told to do so.

——— END OF INSTRUCTIONS ———

Part A: Multiple Choice Questions (Total: 30 marks)

Each multiple choice question (MCQ) is worth two marks and has exactly **one** correct answer.

1. What is the policy for late submission of assignments in CS1231S?

   A. Late submissions are accepted without any penalty.

   B. Late submissions are not accepted.

   C. A make-up assignment will be given to students who submitted late.

   D. Students who submitted late will be referred to the school's disciplinary board.

   E. This is a trick question, as there are no assignments given out in CS1231S.

2. Let $Bird(x)$ means "x is a bird" and $Fly(y)$ means "y can fly". Which of the following is a correct logical statement for "All birds cannot fly"?

   A. $\forall x \left(\sim Fly\big(Bird(x)\big)\right)$

   B. $\forall x \ \big(Bird(x) \wedge \sim Fly(x)\big)$

   C. $\forall x \ \big(Bird(x) \rightarrow \sim Fly(x)\big)$

   D. $\forall x \ \forall y \ \big(Bird(x) \rightarrow \sim Fly(y)\big)$

   E. None of options (A), (B), (C), (D) are correct.

3. Which of the following are true?

   (i) $false \rightarrow true.$

   (ii) $(p \wedge q) \ \rightarrow (p \vee q) \vee \Big(q \wedge \Big(r \vee \big(s \wedge (t \vee \sim u)\big)\Big)\Big).$

   (iii) $(p \vee q \vee \sim r) \wedge \big(s \vee \sim(p \vee q)\big) \rightarrow (s \vee \sim r).$

   A. Only (ii).

   B. Only (iii).

   C. Only (i) and (ii).

   D. Only (ii) and (iii).

   E. None of options (A), (B), (C), (D) are correct.

4. Given the statement form below where p, q, r, s are statement variables,

$$\Big(\Big(\big((p \rightarrow q) \rightarrow (q \rightarrow r)\big) \rightarrow (r \rightarrow s)\Big) \vee p\Big) \wedge \big(p \rightarrow (p \rightarrow q)\big) \wedge p$$

   Which of the following is logically equivalent to the above?

   A. p

   B. $p \vee q$

   C. $p \wedge q$

   D. $p \rightarrow q$

   E. None of options (A), (B), (C), (D) are correct.

5. Consider the premises below which are assumed to be true:

$$p \rightarrow (q \wedge r)$$
$$\sim p \rightarrow \sim r$$
$$\sim p \wedge (r \rightarrow \sim p)$$

Which of the following conclusions would make the argument valid?

(i) $p \rightarrow q$

(ii) $\sim p \vee \sim r$

(iii) $q \rightarrow (p \rightarrow r)$

(iv) $\sim p \wedge \sim q \wedge r$

A. Only (i) and (iii).
B. Only (ii) and (iv).
C. Only (i), (ii) and (iii).
D. Only (i), (ii) and (iv).
E. None of options (A), (B), (C), (D) are correct.

6. Which of the statements below are true for any sets A, B and C? ($\mathcal{P}(X)$ denotes the power set of X.)

(i) $\big(\mathcal{P}(A) \subseteq \mathcal{P}(B)\big) \rightarrow (A \subseteq B)$.

(ii) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

(iii) $\big((A \times B) \subseteq (B \times C)\big) \rightarrow (A \subseteq C)$.

A. Only (i) and (ii).
B. Only (ii) and (iii).
C. Only (i) and (iii).
D. All of (i), (ii) and (iii).
E. None of options (A), (B), (C), (D) are correct.

7. Which of the statements below are true?

(i) If $A = \{1,2,3,4\}$, then $\{\{2,3\}, \{4\}, \{3,2\}, \{1\}\}$ is a partition of A.

(ii) $\{\mathbb{Z}\}$ is a partition of $\mathbb{Z}$.

(iii) $\mathbb{Z}$ is a partition of $\mathbb{Z}$.

A. Only (ii).
B. Only (iii).
C. Only (i) and (ii).
D. Only (ii) and (iii).
E. None of options (A), (B), (C), (D) are correct.

8. Let S be any finite set and $\mathcal{P}(S)$ the power set of S. Which of the statements below are true?

   (i) $|\mathcal{P}(S) \setminus \mathcal{P}(\emptyset)| = 2^{|S|} - 1$.

   (ii) $|S \cap \mathcal{P}(S)| = 0$.

   (iii) $|S| \times |\mathcal{P}(S)| = |S \times \mathcal{P}(S)|$.

   A. Only (i).
   B. Only (iii).
   C. Only (i) and (ii).
   D. Only (i) and (iii).
   E. None of options (A), (B), (C), (D) are correct.

9. Let A be the set of the first 10 prime numbers. Which of the statements below are true?

   (i) There are 10 equivalence classes in the divisibility relation on A.

   (ii) There are 10 minimal elements in the divisibility relation on A.

   A. Only (i).
   B. Only (ii).
   C. Both (i) and (ii).
   D. Neither (i) nor (ii).
   E. This is impossible because a relation on the same set cannot be both an equivalence relation and a partial order.

10. Let R be a binary relation defined on the set {true, false}. For any two logic statements p and q, R is defined as follows:

$$p\ R\ q \iff (p \to q \equiv true)$$

   Which of the following are true?

   (i) R is reflexive.

   (ii) R is symmetric.

   (iii) R is antisymmetric.

   (iv) R is transitive.

   A. Only (i) and (ii).
   B. Only (iii) and (iv).
   C. Only (i), (ii) and (iv).
   D. Only (i), (iii) and (iv).
   E. None of options (A), (B), (C), (D) are correct.

11. Let $A = \{a, b, c\}$ and $R = \{(a, a), (a, b), (b, c), (c, a), (c, b)\}$. What is $R \circ R \circ R$?

    A. R

    B. $A \times A$

    C. $(A \times A) \setminus \{(b, c)\}$

    D. $(A \times A) \setminus \{(a, b), (b, c)\}$

    E. None of options (A), (B), (C), (D) are correct.

12. Let A be any non-empty set and R be any non-empty binary relation on A. Which of the statements below are true?

    (i) $|R \circ R| = |R|$.

    (ii) $|R \circ R| = |R| \times |R|$.

    (iii) $|R \circ R| \neq |R| \times |R|$.

    A. Only (i).

    B. Only (ii).

    C. Only (iii).

    D. Only (ii) and (iii).

    E. None of options (A), (B), (C), (D) are correct.

13. Let $\preccurlyeq$ be a partial order on a non-empty set A. A subset C of A is called a **chain** if and only if every pair of elements in C is comparable, that is, $\forall a, b \in C\ (a \preccurlyeq b \lor b \preccurlyeq a)$. A **maximal chain** is a chain M such that $t \notin M \Rightarrow M \cup \{t\}$ is not a chain. The length of a chain is one less than the number of elements in it.

    Given the set $A = \{2,3,6,7,9,18,24,28\}$ and the divisibility relation on A, how many longest maximal chains are there?

    A. 3

    B. 5.

    C. 7.

    D. 9.

    E. None of options (A), (B), (C), (D) are correct.

14. Let $\preccurlyeq$ be a partial order on a non-empty set A. A subset D of A is called an **antichain** if and only if no two distinct elements in D are comparable, that is, $\forall a, b \in D\ \big(a \neq b \rightarrow {\sim}(a \preccurlyeq b \vee b \preccurlyeq a)\big)$.

    Given the set $A = \{2,3,6,7,9,18,24,28\}$ and the divisibility relation on A, which of the following are antichains of the poset $(A, |)$?

    (i) $\{2, 3, 28\}$
    (ii) $\{7, 9, 24\}$
    (iii) $\{6\}$.

    A. Only (ii).
    B. Only (iii).
    C. Only (i) and (ii).
    D. Only (ii) and (iii).
    E. None of options (A), (B), (C), (D) are correct.

15. Given the set $A = \{2,3,6,7,9,18,24,28\}$ and the divisibility relation on A, which of the following are linearizations $\preccurlyeq^*$ of the relation?

    (i) $2 \preccurlyeq^* 3 \preccurlyeq^* 6 \preccurlyeq^* 7 \preccurlyeq^* 9 \preccurlyeq^* 18 \preccurlyeq^* 24 \preccurlyeq^* 28$
    (ii) $7 \preccurlyeq^* 3 \preccurlyeq^* 2 \preccurlyeq^* 28 \preccurlyeq^* 9 \preccurlyeq^* 6 \preccurlyeq^* 18 \preccurlyeq^* 24$
    (iii) $7 \preccurlyeq^* 2 \preccurlyeq^* 3 \preccurlyeq^* 6 \preccurlyeq^* 18 \preccurlyeq^* 9 \preccurlyeq^* 24 \preccurlyeq^* 28$.

    A. Only (i).
    B. Only (i) and (ii).
    C. Only (i) and (iii).
    D. All of (i), (ii) and (iii).
    E. None of options (A), (B), (C), (D) are correct.

Part B (Total: 20 marks)

16. **Sets [Total: 5 marks]**
    Let $A = \{\emptyset, x\}$. Write out the following sets in set roster notation. Working is not required. Please write clearly. Note that $\mathcal{P}(X)$ denotes the power set of X.

    (a) $\mathcal{P}(A) \setminus \mathcal{P}\big(\mathcal{P}(\emptyset)\big)$ [1 mark]
    (b) $\mathcal{P}\Big(\big(\mathcal{P}(A) \setminus \mathcal{P}(\emptyset)\big) \setminus \{A\}\Big)$ [2 marks]
    (c) $\big(\mathcal{P}(\emptyset) \times \mathcal{P}(A)\big) \setminus \big(\mathcal{P}(A) \times \mathcal{P}(\emptyset)\big)$ [2 marks]

17. **Relations [Total: 15 marks]**

(a) Given $A = \{a, b, c\}$ and $R = \{(a, b), (a, c)\}$ is a binary relation on A. Let R^r, R^s and R^t be the reflexive closure, symmetric closure and transitive closure, respectively, of R. What is the cardinality of R^r? R^s? R^t? [3 marks]

(b) Given $B = \{a, b, c, d\}$ and $S = \{(a, b), (b, a), (b, d), (d, c), (b, b), (d, d)\}$ is a binary relation on B. If $S \cup T$ is the transitive closure of S, and $S \cap T = \emptyset$, what is T? [2 marks]

(c) Given $E = \{2,4,6,8,10,12\}$ and V is the divisibility relation on E, that is,

$$\forall x, y \in E\ (x\ V\ y \Leftrightarrow x|y).$$

Write out the (i) minimal element(s), (ii) maximal element(s), (iii) smallest element, and (iv) largest element. If there is no such element, write "None" instead of leaving it blank. Leaving your answer blank will be taken as no answer. [2 marks]

Definitions: Let $\preccurlyeq$ be a partial order on a set A and let $a, b \in A$.

We say a and b are **comparable** if and only if $a \preccurlyeq b$ or $b \preccurlyeq a$.

We say a and b are **compatible** if and only if there exists $c \in A$ such that $a \preccurlyeq c$ and $b \preccurlyeq c$.

Definitions: Let $\preccurlyeq$ be a partial order on a non-empty set A. A subset C of A is called a **chain** if and only if every pair of elements in C is comparable, that is, $\forall a, b \in C\ (a \preccurlyeq b \vee b \preccurlyeq a)$. A **maximal chain** is a chain M such that $t \notin M \Rightarrow M \cup \{t\}$ is not a chain. The length of a chain is one less than the number of elements in it.

(d) Given the set E and relation V in part (c) above, write out two maximal chains of different lengths. [2 marks]

(e) The binary relations R_1 and R_2, both on $\mathbb{Z}^+$, are defined as follows: $\forall x, y \in \mathbb{Z}^+$,

$$x\, R_1\, y \quad \Leftrightarrow \quad y = 2x$$

$$x\, R_2\, y \quad \Leftrightarrow \quad x = y^2$$

How is x related to y under the relation $(R_1 \circ R_2)$? [2 marks]

(f) Let W be a binary relation on $\mathbb{Z} \times \mathbb{Z}$ defined as follows: $\forall (a, b), (c, d) \in \mathbb{Z} \times \mathbb{Z}$,

$$(a, b)\ W\ (c, d) \quad \Leftrightarrow \quad (a \le c) \wedge (b \ge d).$$

Is the statement below true or false?

$$\forall x, y \in \mathbb{Z} \times \mathbb{Z},\ x \text{ and } y \text{ are comparable.}$$

State "true" or "false", followed by a proof or counterexample. [2 marks]

(g) Given the relation W in part (f) above, is the statement below true or false?

$$\forall x, y \in \mathbb{Z} \times \mathbb{Z},\ x \text{ and } y \text{ are compatible.}$$

State "true" or "false", followed by a proof or counterexample. [2 marks]

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