CS1231S: Discrete Structures

Tutorial #2: Logic of Quantified Statements (Predicate Logic)

(Week 4: 2 – 6 September 2024)

Answers

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1. Discussion Questions

These are meant for you to discuss on Canvas. No answers will be provided.

D1 An elementary definition in number theory is the following on divisibility:

For integers d and n, $d|n$ if and only if $n = kd$ for some integer k.

(Here, $d|n$ means "d divides n", and d is called a *divisor* or *factor*.)

(a) State the above definition symbolically.

(b) According to the above definition, does 2 divide $2\sqrt{2}$?

D2. Explain why English can be ambiguous at times using the following sentence:

"Every boy loves a girl."

Interpret the above sentence in two ways and write the quantified statement for each of the interpretations.

D3. The following table shows when the quantified statements are true and when they are false.

Statement	True when…	False when…
$\forall x\, P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x\, P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Complete the table below for mixed quantifiers.

Statement	True when…	False when…
$\forall x \forall y\, P(x,y)$		
$\forall x \exists y\, P(x,y)$		
$\exists x \forall y\, P(x,y)$		
$\exists x \exists y\, P(x,y)$		

D4. Let the domain of discourse D be the set of all students at NUS, and let $M(s)$ be "s is a Math major", $C(s)$ be "s is a Computer Science major" and $E(s)$ be "s is an Engineering major". Express each of the following statements using quantifiers, variables and the predicates $M(s)$, $C(s)$ and $E(s)$. Part (a) has been done for you.

(a) Every Computer Science major is an Engineering major.

Answer: $\forall s \in D\ \big(C(s) \rightarrow E(s)\big)$.

Discuss: Why is the following answer wrong?

Wrong answer: $\forall s \in D\ \big(C(s) \wedge E(s)\big)$.

Can you give an example to show the difference between these two answers?

(b) No Computer Science major are Engineering majors.

(c) Some Computer Science major are not Math majors.

(d) If a student is not a Math major, then the student is either a Computer Science major or an Engineering major, but not both.

2. Common Mistakes

- Using commas (,) in place of appropriate connectives, for example, $\forall x\ P(x), Q(x)$ where it should be $\forall x\ \big(P(x) \wedge Q(x)\big)$. Note that the comma does <u>not</u> represent conjunction, disjunction, or any logical connective.
- Treating predicates as functions returning some value. Eg: given the following predicates
  - $Loves(x, y)$: x loves y
  - $Reindeer(x)$: x is a reindeer

  Some students wrote $Loves\big(x, Reindeer(y)\big)$ in part of their answers. Since $Reindeer(y)$ is a predicate, its value is either **true** or **false**. So, the above is akin to writing $Loves(x, true)$ or $Loves(x, false)$ which does not make sense! The correct way is to use the appropriate connectives, eg: $Reindeer(y) \wedge Loves(x, y)$.

3. Additional Notes

Note that "logic of quantified statements" (chapter 3) is commonly known as "predicate logic", as opposed to "propositional logic" in chapter 2.

We picked up some frequently asked questions and created this *Additional Notes* section to include some materials not covered in lecture that might be of interest to you.

Equivalent expressions: The following quantified statements are equivalent. We use the shorter notation on the left.

- $\forall x \in D, P(x) \equiv \forall x\ \big((x \in D) \rightarrow P(x)\big)$
- $\exists x \in D, P(x) \equiv\ \exists x\ \big((x \in D) \wedge P(x)\big)$

Sometimes, the comma is omitted, eg: $\forall x \in D\ P(x)$.

Well-formed formula (wff)

- **true** and **false** are wffs.
- A proposition variable (eg: x, p) is a wff.
- A predicate name followed by a list of variables (eg: $P(x)$, $Q(x,y)$), which is called an *atomic formula*, is a wff.
- If A, B and C are wffs, then so are $\sim A$, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$ and $(A \leftrightarrow B)$.
- If x is a proposition variable and A is a wff, then so are $\forall x\ A$ and $\exists x\ A$.

Bound variables, Scope of quantifiers, and the use of Parentheses

- When a quantifier is used on a variable x in a predicate statement, we say that variable x is *bound*. If no quantifier is used on a variable, we say that the variable x is *free*.

  Examples: In $\forall x\ \exists y\ P(x,y)$, both x and y are bound. In $\forall x\ P(x,y)$, x is bound but y is free.
- The scope of a quantifier is the range in the formula where the quantifier "engages in". It is put right after the quantifier, often in parentheses (eg: $\forall x\ \big(P(x)\big)$).

  Sometimes, when parentheses are not present (eg: $\forall x\ P(x)$), the scope is understood to be the smallest wff following the quantification.
- For example, in $\exists x\ P(x,y)$, the variable x is bound while y is free. In $\forall x\big(\exists y\ P(x,y) \vee Q(x,y)\big)$, x and the red y in $P(x,y)$ are bound, but the blue y in $Q(x,y)$ is free, because the scope of $\exists y$ is only $P(x,y)$ (the smallest wff), whereas the scope of $\forall x$ is $\big(\exists y\ P(x,y) \vee Q(x,y)\big)$. If you want to change the blue y to red y, you need to add parentheses: $\forall x\ \Big(\exists y\ \big(P(x,y) \vee Q(x,y)\big)\Big)$. Then, the outermost pair of parentheses may be removed, i.e.: $\forall x\ \exists y\ \big(P(x,y) \vee Q(x,y)\big)$.

4. Tutorial Questions

1. For each of the following statements, write its **converse**, **inverse** and **contrapositive**. Indicate which among the statement, its converse, its inverse, and its contrapositive are true and which are false. Give a counterexample for each that is false. Proof not required if it is true. The predicate $Even(n)$ means that n is an even integer.

   a. $\forall n \in \mathbb{Z}\ (6|n \rightarrow 2|n \wedge 3|n)$. (Note: "$d|n$" is as defined in D1.)

   b. $\forall x\ (x \in \mathbb{Q} \rightarrow x \in \mathbb{Z})$.

   c. $\forall p, q \in \mathbb{Z}\ \big(Even(p) \wedge Even(q) \rightarrow Even(p+q)\big)$.

   Answers:

   a. Statement: $\forall n \in \mathbb{Z}\ (6|n \rightarrow 2|n \wedge 3|n)$. (True)
      Converse: $\forall n \in \mathbb{Z}\ (2|n \wedge 3|n \rightarrow 6|n)$. (True)
      Inverse: $\forall n \in \mathbb{Z}\ (6 \nmid n \rightarrow 2 \nmid n \vee 3 \nmid n)$. (True)
      Contrapositive: $\forall n \in \mathbb{Z}\ (2 \nmid n \vee 3 \nmid n \rightarrow 6 \nmid n)$. (True)

   b. Statement: $\forall x\ (x \in \mathbb{Q} \rightarrow x \in \mathbb{Z})$. (False)
      Converse: $\forall x\ (x \in \mathbb{Z} \rightarrow x \in \mathbb{Q})$. (True)
      Inverse: $\forall x\ (x \notin \mathbb{Q} \rightarrow x \notin \mathbb{Z})$. (True)
      Contrapositive: $\forall x\ (x \notin \mathbb{Z} \rightarrow x \notin \mathbb{Q})$. (False)
      Counterexample for original statement and contrapositive: Let $x = 1/2$.

   c. Statement: $\forall p, q \in \mathbb{Z}\ \big(Even(p) \wedge Even(q) \rightarrow Even(p+q)\big)$. (True)
      Converse: $\forall p, q \in \mathbb{Z}\ \big(Even(p+q) \rightarrow Even(p) \wedge Even(q)\big)$. (False)
      Inverse: $\forall p, q \in \mathbb{Z}\ \big({\sim}Even(p) \vee {\sim}Even(q) \rightarrow {\sim}Even(p+q)\big)$. (False)
      Contrapositive: $\forall p, q \in \mathbb{Z}\ \big({\sim}Even(p+q) \rightarrow {\sim}Even(p) \vee {\sim}Even(q)\big)$. (True)
      Counterexample for converse and inverse: Let $p = q = 1$.

2. A long time ago, you already knew the following:

   a. There is no biggest number, i.e. no matter how big a number is, there is always another number that is bigger.

   b. Given any two distinct numbers, you can always find another number between them.

   Formulate these two ideas symbolically, using quantified statements. What are the domains of the numbers that make the above statements true?

   Answers:
   Students may pick any appropriate domain.

   a. ${\sim}\exists x \in \mathbb{N}\ \forall y \in \mathbb{N}\ (y \leq x)$, or, equivalently, $\forall x \in \mathbb{N}\ \exists y \in \mathbb{N}\ (y > x)$.
      Nonnegative integers $\mathbb{N}$ here can be replaced by integers $\mathbb{Z}$, or rational numbers $\mathbb{Q}$, or real numbers $\mathbb{R}$ (but not complex numbers $\mathbb{C}$).

   b. $\forall x \in \mathbb{Q}\ \forall y \in \mathbb{Q}\ \big(x < y \rightarrow \exists z \in \mathbb{Q}\ (x < z) \wedge (z < y)\big)$.
      $\mathbb{Q}$ here can be replaced by $\mathbb{R}$, but not by $\mathbb{Z}$ or $\mathbb{C}$.
      (Note: $\forall x \in \mathbb{Q}\ \forall y \in \mathbb{Q}$ can be shortened to $\forall x, y \in \mathbb{Q}$.)

3. Later in this course you will learn "relations". For a binary relation R on a set A, there are three important properties:

   a. R is **reflexive** if and only if "xRx for any x in A".
   b. R is **symmetric** if and only if "for every x and y in A, if xRy then yRx".
   c. R is **transitive** if and only if "for all x, y and z in A, if xRy and yRz, then xRz".

   For each property, rewrite the condition in "…" symbolically, using quantified statements with logical connectives instead of words such as "for all, and, if then".

   Answers:

   a. R is reflexive if and only if $\forall x \in A\ (xRx)$.
   b. R is symmetric if and only if $\forall x, y \in A\ (xRy \rightarrow yRx)$.
   c. R is transitive if and only if $\forall x, y, z \in A\ (xRy \wedge yRz \rightarrow xRz)$.

   Here, "for any", "for every", "for all" mean the same thing.

4. Recall the definition of rational numbers (Lecture 1 slide 37):

   $$r \text{ is rational} \Leftrightarrow \exists a, b \in \mathbb{Z} \text{ s.t. } r = \frac{a}{b} \text{ and } b \neq 0.$$

   Prove or disprove the following statements:

   a. Integers are closed under division.
   b. Rational numbers are closed under addition.
   c. Rational number are closed under division.

   Answers:

   a. **False.** Counterexample: Let $a = 3, b = 2$, then $\frac{a}{b} = 1.5$ is not an integer.

   b. **True.**
      1. Let r and s be arbitrarily chosen rational numbers.
      2. Then $\exists a, b, c, d \in \mathbb{Z}$ s.t. $r = \frac{a}{b}, s = \frac{c}{d}$ and $b \neq 0, d \neq 0$. (by defn of rational numbers)
      3. Hence, $r + s = \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ (by basic algebra)
      4. $ad + bd \in \mathbb{Z}$ and $bd \in \mathbb{Z}$. (closure of integers under $+$ and $\times$)
      5. Moreover, $bd \neq 0$ since $b \neq 0, d \neq 0$. (Appendix A T11 Zero Product Property) (Note: this is important! Some students missed this.)
      6. Hence, $r + s$ is rational. (by definition of rational numbers)
      7. Therefore, rational numbers are closed under addition.

   c. **False.** Counterexample: Let $r, s \in \mathbb{Q}$ and $s = 0$. Then r/s is undefined.

5. Given $A = \{1,3,5,7,11,13\}$ and $B = \{0,2,4,6\}$, for each of the statements (a) to (j), explain whether the statement is true or false.

a. $\forall x \in B\ \forall y \in B\ (x - y \in B)$

b. $\forall x \in A\ \forall y \in A\ \big((x < y) \wedge (y < 10) \rightarrow y - x \in B\big)$

c. $\forall x \in A\ \forall y \in B\ (x = y + 1)$

d. $\exists x \in A\ \exists y \in B\ (x = y + 1)$

e. $\exists y \in B\ \exists x \in A\ (x = y + 1)$

f. $\forall x \in A\ \forall y \in B\ (x \neq y + 1)$

g. $\exists x \in A\ \exists y \in B\ (x \neq y + 1)$

h. $\forall x \in A\ \exists y \in B\ (x > y)$

i. $\forall x \in A\ \exists y \in B\ (x \leq y)$

j. $\exists y \in B\ \forall x \in A\ (x > y)$

Answers:

a. $\forall x \in B\ \forall y \in B\ (x - y \in B)$
False. Counterexample: $x = 2, y = 4$.

b. $\forall x \in A\ \forall y \in A\ \big((x < y) \wedge (y < 10) \rightarrow y - x \in B\big)$
True.
Case $x = 1, y = 3$: $y - x = 2 \in B$.
Case $x = 1, y = 5$: $y - x = 4 \in B$.
Case $x = 1, y = 7$: $y - x = 6 \in B$.
Case $x = 3, y = 5$: $y - x = 2 \in B$.
Case $x = 3, y = 7$: $y - x = 4 \in B$.
Case $x = 5, y = 7$: $y - x = 2 \in B$.

c. $\forall x \in A\ \forall y \in B\ (x = y + 1)$
False. Counterexample: $x = 1, y = 2$.

d. $\exists x \in A\ \exists y \in B\ (x = y + 1)$
True. Example: $x = 3, y = 2$.

e. $\exists y \in B\ \exists x \in A\ (x = y + 1)$
True. [(e) is equivalent to (d).]

f. $\forall x \in A\ \forall y \in B\ (x \neq y + 1)$
False. Counterexample: $x = 3, y = 2$. [(f) is the negation of (d).]

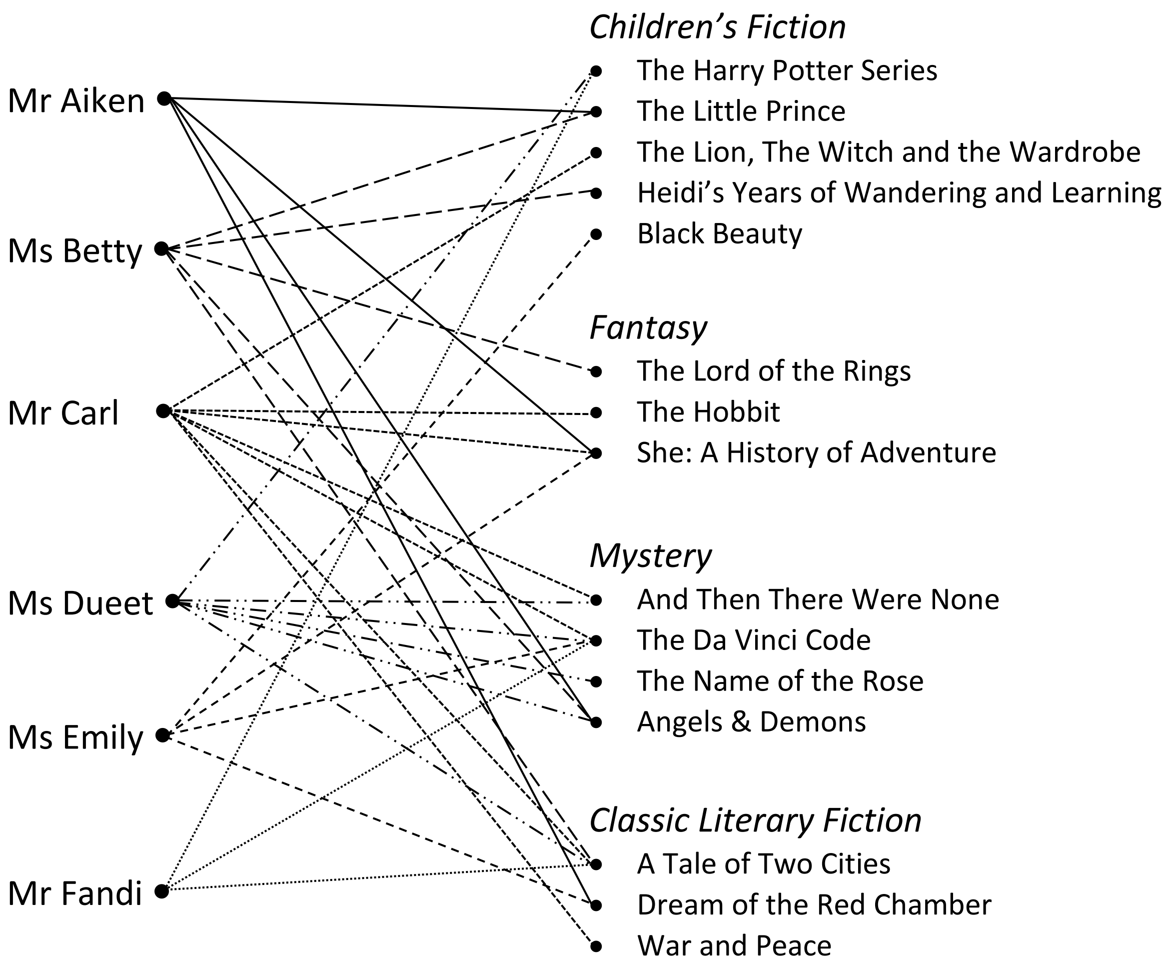
g. $\exists x \in A\ \exists y \in B\ (x \neq y + 1)$
True. Example: $x = 1, y = 2$. [(g) is the negation of (c).]

h. $\forall x \in A\ \exists y \in B\ (x > y)$
True. We can pick $y = 0$, so every element in A is larger than 0.

i. $\forall x \in A\ \exists y \in B\ (x \leq y)$
False. Counterexample: $x = 11$.

j. $\exists y \in B\ \forall x \in A\ (x > y)$
True. Example: $y = 0$.

6. Refer to the figure below, which shows six readers and four book genres (Children's Fiction, Fantasy, Mystery and Classic Literary Fiction) with selected book titles of each genre. A line is drawn between a reader and a book title if, and only if, that reader reads that book. For example, Mr Aiken reads "The Little Prince", but Ms Dueet does not read "Black Beauty".



For each of the following, indicate whether the statement is true or false and explain why. (You are not required to write the quantified statements.)

a. Some title is read by all the female readers.
b. Every reader reads some title in every genre.
c. Some reader reads all titles of some genre.
d. There is some genre for which some reader does not read any of its titles.

Answers:

a. **False.** None of the titles is read by all three female readers.

b. **False.** Ms Dueet doesn't read any Fantasy title.

c. **True.** Ms Dueet reads all the Mystery titles.

d. **True.** None of the Fantasy titles is read by Ms Dueet/Mr Fandi.

7. This question illustrates that one can "prove" anything, including nonsense, using bad logic.

   a. The following is a proof for: $\forall x \in \mathbb{R}\ (x^2 \geq 0)$. What is wrong with this "proof"?

      "There are 3 cases to consider: $x < 0, x = 0$ and $x > 0$. If $x < 0$, for example, $x = -3$, then $x^2 = 9 \geq 0$; if $x = 0$, then $x^2 = 0$; if $x > 0$, say $x = 4$, then $x^2 = 16 \geq 0$. Therefore, in all cases, $x^2 \geq 0$."

   b. Use the same logic in (a) to prove: $\forall x \in \mathbb{R}\ (x^3 = x)$.

   c. The following is another proof for: $\forall x \in \mathbb{R}\ (x^2 \geq 0)$. What is wrong with this "proof"?

      "Prove by contradiction. Suppose $x^2 < 0$ for all real numbers x. Let $x = 3$, then $x^2 = 9 > 0$ which is a contradiction. Therefore, $\forall x \in \mathbb{R}\ (x^2 \geq 0)$."

   d. Use the same logic in (c) to prove: $\forall x \in \mathbb{R}\ (x^3 = x)$.

Answers:

a. This looks like proof by division into cases, but it is really a "proof by examples", which is invalid for a universal statement. Error: it does not consider arbitrary real number x.

b. Claim: $\forall x \in \mathbb{R}\ (x^3 = x)$.

   "There are 3 cases to consider: $x < 0, x = 0$ and $x > 0$. If $x < 0$, for example, $x = -1$, then $x^3 = -1 = x$; if $x = 0$, then $x^3 = 0 = x$; if $x > 0$, say $x = 1$, then $x^3 = 1 = x$. Therefore, in all cases, $x^3 = x$."

c. To prove by contradiction, one must consider $\sim\big(\forall x \in \mathbb{R}\ (x^2 \geq 0)\big)$, which is $\exists x \in \mathbb{R} \sim(x^2 \geq 0)$. What the proof considers is $\forall x \in \mathbb{R} \sim(x^2 \geq 0)$.

d. Claim: $\forall x \in \mathbb{R}\ (x^3 = x)$.

   "Prove by contradiction. Suppose $x^3 \neq x$ for all real numbers x. Let $x = 0$, then $x^3 = 0 = x$, which is a contradiction. Therefore, $\forall x \in \mathbb{R}\ (x^3 = x)$."

8. The following is a partial proof of the claim:

$$\forall x \in \mathbb{R}\left((x^2 > x) \rightarrow (x < 0) \vee (x > 1)\right).$$

1.	Let r be an arbitrarily chosen real number.
2.	Suppose $r^2 > r$.
	2.1. Then $r^2 - r > 0$, or $r(r-1) > 0$. (by basic algebra)
	2.2. So, both r and $r-1$ are positive, or both are negative. (by Appendix A, T25)
	2.3. …
3.	Therefore, $\forall x \in \mathbb{R}\left((x^2 > x) \rightarrow (x < 0) \vee (x > 1)\right)$.

Note: Some students drew diagrams (eg: graphs) and used them as proofs. In this module, do not use diagrams for proofs, unless otherwise instructed. If you use diagrams, you need to explain the diagrams.

a. In step 2, we explore the case $r^2 > r$. Do we need to include the case $r^2 \leq r$? Why?

b. Complete the proof.

c. Step 3 is an application of **universal generalization**. Explain what it means.

Answers:

a. A conditional statement $p \rightarrow q$ is always true when the hypotheses/antecedent p is false, therefore, there is no need to examine the case $r^2 \leq r$.

b.

1. Let r be an arbitrarily chosen real number.
2. Suppose $r^2 > r$.
   - 2.1. Then $r^2 - r > 0$, or $r(r-1) > 0$. (by basic algebra)
   - 2.2. So, both r and $r-1$ are positive, or both are negative. (by Appendix A, T25)
   - 2.3. Case 1: Both r and $r-1$ are positive.
     - 2.3.1. $(r > 0) \wedge (r - 1 > 0) \rightarrow (r > 1)$.
   - 2.4. Case 2: Both r and $r-1$ are negative.
     - 2.4.1 $(r < 0) \wedge (r - 1 < 0) \rightarrow (r < 0)$.
   - 2.5. From lines 2.3.1 and 2.4.1, we have $(r > 1) \vee (r < 0)$.
   - 2.6. Therefore, $(r^2 > r) \rightarrow (r > 1) \vee (r < 0)$.
3. Therefore, $\forall x \in \mathbb{R}\left((x^2 > x) \rightarrow (x < 0) \vee (x > 1)\right)$.

c. Let predicates $P(x)$ be "$x^2 > x$" and $Q(x)$ be "$(x < 0) \vee (x > 1)$". Since we have established that for an arbitrarily chosen real number r, $P(r) \rightarrow Q(r)$, we may generalize it to all real numbers, i.e., $\forall x \in \mathbb{R}\left(P(x) \rightarrow Q(x)\right)$.

9. Let V be the set of all visitors to Universal Studios Singapore on a certain day, $T(v)$ be "v took the Transformers ride", $G(v)$ be "v took the Battlestar Galactica ride", $E(v)$ be "v visited the Ancient Egypt", and $W(v)$ be "v watched the Water World show".

   Express each of the following statements using quantifiers, variables, and the predicates $T(v)$, $G(v)$, $E(v)$ and $W(v)$. The statements are not related to one another. Part (a) has been done for you.

   a. Every visitor watched the Water World show.

Answer for (a): $\forall v \in V\ (W(v))$.

   b. Every visitor who took the Battlestar Galactica ride also took the Transformers ride.

   c. There is a visitor who took both the Transformers ride and the Battlestar Galactica ride.

   d. No visitor who visited the Ancient Egypt watched the Water World show.

   e. Some visitors who took the Transformers ride also visited the Ancient Egypt but some (who took the Transformers ride) did not (visit the Ancient Egypt).

Answers

b. $\forall v \in V\ (G(v) \rightarrow T(v))$

c. $\exists v \in V\ (T(v) \wedge G(v))$

d. $\forall v \in V\ (E(v) \rightarrow {\sim}W(v))$
   Alternatively: $\forall v \in V\ ({\sim}E(v) \vee {\sim}W(v))$

e. $\big(\exists v \in V\ (T(v) \wedge E(v))\big) \wedge \big(\exists u \in V\ (T(u) \wedge {\sim}E(u))\big)$

   Note that the above is a conjunction (AND) of two existential clauses. Each existential variable v or u has a scope only within its own clause. Therefore, there is no ambiguity to use the same variable in both clauses, as shown below:

   $$\big(\exists v \in V\ (T(v) \wedge E(v))\big) \wedge \big(\exists v \in V\ (T(v) \wedge {\sim}E(v))\big)$$

   Nevertheless, you may use different variable names to avoid confusion.

   Also, note that it is incorrect to write

   $$\exists v \in V\ \big((T(v) \wedge E(v)) \wedge (T(v) \wedge {\sim}E(v))\big)$$

   because the scope of v here covers both $(T(v) \wedge E(v))$ as well as $(T(v) \wedge {\sim}E(v))$. This means that if there exists a visitor who took the Transformers Ride and visited Ancient Egypt, the <u>same visitor</u> took the Transformers Ride but did not visit Ancient Egypt! The statement becomes false (by applying commutativity and associativity of conjunction).

10. Given the following argument:

1. If an object is above all the triangles, then it is above all the blue objects. 2. If an object is not above all the gray objects, then it is not a square. 3. Every black object is a square. 4. Every object that is above all the gray objects is above all the triangles.
$\therefore$ If an object is black, then it is above all the blue object.

a. Reorder the premises in the argument to show that the conclusion follows as a valid consequence from the premises, by applying universal transitivity (Lecture 3 slide 93). (Hint: It may be helpful to rewrite the statements in if-then form and replace some statements by their contrapositives.)

   You may use self-explanatory predicate names such as $Triangle(x)$, $Square(x)$, etc.

b. Rewrite your answer in part (a) using predicates and quantified statements.

Answers:

a.

3. If an object is black, then it is a square.
2. (Contrapositive form) If an object is a square, then it is above all the gray objects.
4. If an object is above all the gray objects, then it is above all the triangles.
1. If an object is above all the triangles, then it is above all the blue objects.

$\therefore$ If an object is black, then it is above all the blue objects.

b.

Let O, the domain, be the set of objects.

3. $\forall x \in O\ (Black(x) \rightarrow Square(x))$.
2. (Contrapositive form) $\forall x \in O\ \Big(Square(x) \rightarrow \big(\ \forall y \in O(Gray(y) \rightarrow Above(x,y))\big)\Big)$.
4. $\forall x \in O\ \Big(\big(\forall y \in O\ (Gray(y) \rightarrow Above(x,y))\big) \rightarrow \big(\forall z \in O\ (Triangle(z) \rightarrow Above(x,z))\big)\Big)$.
1. $\forall x \in O\ \Big(\big(\forall z \in O\ (Triangle(z) \rightarrow Above(x,z))\big) \rightarrow \big(\forall w \in O\ (Blue(w) \rightarrow Above(x,w))\big)\Big)$.

$\therefore \forall x \in O\ \Big(Black(x) \rightarrow \big(\ \forall w \in O\ (Blue(w) \rightarrow Above(x,w))\big)\Big)$.

11. [Past year's midterm test question]

Prove that if n is a product of two positive integers a and b, then $a \leq n^{1/2}$ or $b \leq n^{1/2}$.

Answers:

Proof by contraposition. (Recall: $p \rightarrow q \equiv {\sim}q \rightarrow {\sim}p$)

1. The contrapositive of the given statement is:
   If $a > n^{1/2}$ and $b > n^{1/2}$, then n is not a product of a and b. (by De Morgan's law)
2. Suppose $a > n^{1/2}$ and $b > n^{1/2}$, then $ab > n^{1/2} \cdot n^{1/2} = n$. (by Appendix A T27)
3. Since $ab \neq n$, the contrapositive statement is true.
4. Therefore, the original statement is true.

Or, using proof by contradiction (Recall that ${\sim}(p \rightarrow q) \equiv p \wedge {\sim}q$).)

1. Suppose not (that is, taking the negation of the given statement), we have
   $n = ab$ and $a > n^{1/2}$ and $b > n^{1/2}$. (by De Morgan's law)
2. Since $a > n^{1/2}$ and $b > n^{1/2}$, we have $ab > n^{1/2} \cdot n^{1/2} = n$. (by Appendix A T27)
3. This contradicts $n = ab$ in line 1.
4. Therefore, the original statement is true.

Note: Appendix A is given in Canvas as well as CS1231S "Lectures" page.

Appendix A T27.

$\forall a, b, c \in \mathbb{R}$, if $0 < a < c$ and $0 < b < d$, then $0 < ab < cd$.