MA 5219 - Logic and Foundations of Mathematics 1
Homework due in Week 3, Tuesday.

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Hand in each starred homework; 1 mark per homework (if it is correct), up to 10 marks in total for homework.

3.1 Conjunctive and Disjunction Normalform. Make formulas in conjunctive normal form and disjunctive normal form which state that exactly 2 of the atoms p_1, p_2, p_3, p_4 are true.

3.2* Worlds. Recall that a world is an entity which assigns a truth-value to every atom. $W \models A$ means that the world makes the formula A true and $W \models X$ means that the world makes all formulas in the set X of formulas true. $X \models A$ means that every world which makes all formulas in X true, also makes the formula A true.
Let V and W be two different worlds, let $X = \{A : V \models A \text{ or } W \models A\}$ and let $Y = \{A \in X : \forall B \in X\, [A \wedge B \in X]\}$. Show the following.
(a) $X \models A$ for every formula A.
(b) There are formulas $A, B \in X$ with $A \wedge B \notin X$.
(c) If $A, B, C \in X$ then at least one of the formulas $A \wedge B$, $A \wedge C$, $B \wedge C$ is in X.
(d) If $Y \models A$ then $A \in Y$.

3.3 Logical Implication. (a) Assume that $W \models X$ for all worlds W which make only finitely many atoms true. Show that $W \models X$ for all worlds and that X contains only tautologies.
(b) Construct a set X of formulas such that $W \models X$ is true iff W makes at most two atoms true and all others false.

3.4 Proof System. Assume that only $(A + B)$ is permitted to connect formulas A, B, which are built from the atoms $p_0, p_1, \ldots$ and the logical constants 0 and 1. Is there a set of rules which is permits to prove A from X whenever $X \models A$ and X is a set of formulas of the above form and A is a formula of the above form? If so, give the set of rules; if not, explain why a set of such rules cannot exist.