MA 5219 - Logic and Foundations of Mathematics 1
Course-Webpage `http://www.comp.nus.edu.sg/~fstephan/mathlogic.html`
Homework due in Week 6, Tuesday 17 September 2013.

Frank Stephan. Departments of Mathematics and Computer Science,
10 Lower Kent Ridge Road, S17#07-04 and 13 Computing Drive, COM2#03-11,
National University of Singapore, Singapore 119076.
Email fstephan@comp.nus.edu.sg
Telephone office 65162759 and 65164246
Office hours Thursday 14.00-15.00h S17#07-04

Hand in each homework which you want to be checked; 1 mark per each correct starred homework; up to 10 marks in total for homework - there will be more than 10 starred homeworks, so you have several chances to try.

6.1* **Programs.** Write programs which compute the following functions using either the given language of the lecture or the language with for-loops, while-loops, if-statements and terms using addition and multiplication with constants. The programs might be permitted to call other subprograms (to be defined as well) or themselves. Input and output are always natural numbers.
(a) The function $\mathit{Choose}(m, n)$ being the coefficient for x^m in the expansion of $(x+1)^n$. The output is 0 in the case that $m > n$.
(b) The function $\mathit{Log3}(n)$ being the maximum m such that $3^m \leq n$; with the exception handling that $\mathit{Log}3(0)$ is 0.
(c) A function $\mathit{Is3Power}(n)$ which is 1 in the case that n is a power of 3 and which is 0 if this not a case. Here $\mathit{Is3Power}(0)$ is 0, that is, $3^{-\infty}$ does not count as a power. However, 3^0 counts as a power of 3.

6.2* **Formulas.** Here show that for the following functions $f_k(x)$ the relation $f_k(x) = y$ can be defined using a formula $\phi_k(x, y)$ in which all quantifiers are bounded. The underlying structure is the set of natural numbers with addition and multiplication. Here a quantifier is bounded if it ranges only over numbers below a term expressed in x and y and constants.
(a) $f_1(x) = y$ iff $y > x$ and y is the least upper bound of x such that y and $y + 2$ are both prime numbers.
(b) $f_2(x) = y$ iff y is even and $y \geq x$ and y is not the sum of two prime numbers.
(c) $f_3(x) = y$ iff $y > x$ and $y = a^5 + b^5$ and $y = c^5 + d^5$ for positive integers a, b, c, d with $\{a, b\} \neq \{c, d\}$.
Is it known that any of these functions is total? That is, is $\forall x \exists y\, [\phi_k(x, y)]$ known to be true or false for any k?

6.3 Definability. Give an easy mathematical condition describing the following set A: $x \in A \Leftrightarrow \forall u, v, w, y\, [x \neq u^2 + v^2 + w^2 + 8 \cdot y]$.