BETWEEN Σ_1- AND Σ_2-INDUCTION

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Abstract. In our work (Chong, Slaman and Yang [CSY2014]) of reverse mathematics on Stable Ramsey's Theorem for pairs, we came across an axiom which we named BME_1. At that time, it was just a technical tool to make things work. Later Kreuzer and Yokoyama [KY2016] showed that BME_1 actually is equivalent to several other axioms that people had studied long before. For example, BME_1 is equivalent to $P\Sigma_1$ which appeared in the standard reference books like Hájek and Pudlák [HP1993]. $P\Sigma_1$ is also located between $I\Sigma_1$ and $I\Sigma_2$ but is incomparable with $B\Sigma_2$. Furthermore, $B\Sigma_2 + P\Sigma_1 \not\Rightarrow I\Sigma_2$.

Further applications of $P\Sigma_1$ was discovered. We (Chong, W. Li, W. Wang and Yang [CLWY2020]) showed that TT^1, which is a tree version of Ramsey's Theorem for singletons, is Π^1_1-conservative over $\mathsf{RCA}_0 + B\Sigma_2 + P\Sigma_1$, consequently, TT^1 does not imply $I\Sigma_2$. Recently, it was further strengthened by Chong, W. Wang and Yang [CWY202?] showing that TT^1 is Π^0_3-conservative over RCA_0, thus, TT^1 does not imply $P\Sigma_1$. These latter results are analogues of Patey and Yokoyama's work [PY2018] saying that RT^2_2 (Ramsey's Theorem for pairs) has the same properties.

If time permits, I will talk about some ongoing projects with many colleagues. For instance, with Chong and W. Wang we showed that $P\Sigma_1 + \Sigma_2$-$\mathsf{WPHP} \not\Rightarrow B\Sigma_2$, where Σ_2-WPHP is a weaker version of the Pigeonhole Principle, introduced in Belanger, Chong, W. Wang, T. Wong and Yang [BCWWY2021] with the property that $B\Sigma_2 \Rightarrow \Sigma_2$-$\mathsf{WPHP} \Rightarrow I\Sigma_1$.

This talk was based on an earlier online talk in Nankai Logic Colloquium.

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