Problem 1: Natural Deduction in Tree Form, 20 points

Prove the following sequents using natural deduction (proof trees):
1) $\vdash (p \wedge q) \rightarrow (p \vee q)$ (5 points)
2) $\vdash (p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$ (5 points)
3) $\vdash (\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)$ (10 points)

Problem 2: Natural Deduction, 45 points

Prove the following sequents using natural deduction (3-column format):

1) $\vdash (p \wedge q) \rightarrow (p \vee q)$ (5 points)
2) $\vdash P \rightarrow (Q \rightarrow P)$ (5 points)
3) $\vdash \neg(P \wedge \neg P)$ (5 points)
4) $\vdash (p \rightarrow q) \rightarrow (r \rightarrow q) \rightarrow (p \vee r) \rightarrow q$ (5 points)
5) $\vdash (p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$ (5 points)
6) $\vdash (\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)$ (10 points)
7) $\vdash ((P \vee Q) \wedge (\neg P \vee \neg Q)) \rightarrow ((P \wedge \neg Q) \vee (\neg P \wedge Q))$ (10 points)

Problem 3: Intuitionistic Logic, 30 points

We can form *intuitionistic logic* by removing the proof rule for double negation elimination: ¬¬E, as well as all proofs that rely on this rule (such as the law of excluded middle). Use **only** Intuitionistic Logic here; **do not use** ¬¬E or any lemmas that use ¬¬E such as LEM. If you are unsure if a lemma uses ¬¬E (*e.g.*, MT?), the safest thing is to only use the basic axioms.
1) $\vdash (\neg\neg\neg P) \rightarrow (\neg P)$ (15 points)
2) $\neg\big((\neg P \rightarrow P) \wedge (P \rightarrow \neg P)\big)$ (15 points)
Note that this problem is quite easy in *classical logic*—that is, **with** ¬¬E or LEM. We will have more to say about intuitionistic logic later in the course.

Problem 4: Semantics, 25 points

Classify the following formulas as valid, satisfiable, or not satisfiable, and prove using the semantic method (truth tables).

1) $(p \wedge q) \rightarrow (p \vee q)$ (5 points)
2) $(p \wedge \neg q) \vee q$ (5 points)
3) $(\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)$ (5 points)
4) $\neg q \rightarrow q$ (5 points)
5) $(\neg q \rightarrow q) \wedge (q \rightarrow \neg q)$ (5 points)