# 02—Traditional Logic

CS 3234: Logic and Formal Systems

Martin Henz and Aquinas Hobor

August 19, 2010

Generated on Thursday 19th August, 2010, 15:26

1. Categorical Terms and their Meaning
2. Propositions, Axioms, Lemmas, Proofs
3. Manipulating Terms and Propositions
4. Arguments and Syllogisms

1. Categorical Terms and their Meaning
   - Origins and Goals
   - Form, not Content
   - Categorical Terms
   - Meaning through models
2. Propositions, Axioms, Lemmas, Proofs
3. Manipulating Terms and Propositions
4. Arguments and Syllogisms

# Traditional Logic

## Origins

Greek philosopher Aristotle (384–322 BCE) wrote treatise *Prior Analytics*; considered the earliest study in formal logic; widely accepted as the definite approach to deductive reasoning until the $19^{\text{th}}$century.

# Traditional Logic

## Origins

Greek philosopher Aristotle (384–322 BCE) wrote treatise *Prior Analytics*; considered the earliest study in formal logic; widely accepted as the definite approach to deductive reasoning until the $19^{\text{th}}$century.

## Goal

Express relationships between sets; allow reasoning about set membership

# Example 1

*All humans are mortal.*
*All Greeks are humans.*
*Therefore, all Greeks are mortal.*

# Example 1

*All humans are mortal.*
*All Greeks are humans.*
*Therefore, all Greeks are mortal.*

Makes “sense”, right?

# Example 1

*All humans are mortal.*
*All Greeks are humans.*
*Therefore, all Greeks are mortal.*

Makes “sense”, right?

Why?

# Example 2

*All cats are predators.*
*Some animals are cats.*
*Therefore, all animals are predators.*

# Example 2

*All cats are predators.*
*Some animals are cats.*
*Therefore, all animals are predators.*

Does not make sense!

# Example 2

*All cats are predators.*
*Some animals are cats.*
*Therefore, all animals are predators.*

Does not make sense!

Why not?

# Example 3

*All slack track systems are caterpillar systems.*
*All Christie suspension systems are slack track systems.*
*Therefore, all Christie suspension systems are caterpillar systems.*

# Example 3

*All slack track systems are caterpillar systems.*
*All Christie suspension systems are slack track systems.*
*Therefore, all Christie suspension systems are caterpillar systems.*

Makes sense, even if you do not know anything about suspension systems.

# Example 3

*All slack track systems are caterpillar systems.*
*All Christie suspension systems are slack track systems.*
*Therefore, all Christie suspension systems are caterpillar systems.*

Makes sense, even if you do not know anything about suspension systems.

**Form, not content**

In logic, we are interested in the form of valid arguments, irrespective of any particular domain of discourse.

# Categorical Terms

## Terms refer to sets

Term `animals` refers to the set of animals,
term `brave` refers to the set of brave persons, etc

# Categorical Terms

**Terms refer to sets**

Term `animals` refers to the set of animals,
term `brave` refers to the set of brave persons, etc

**`Terms`**

The set `Term` contains all terms under consideration

# Categorical Terms

**Terms refer to sets**

Term `animals` refers to the set of animals,
term `brave` refers to the set of brave persons, etc

**`Terms`**

The set `Term` contains all terms under consideration

**Examples**

$$\texttt{animals} \in \texttt{Term}$$

$$\texttt{brave} \in \texttt{Term}$$

# Models

## Meaning

A model $\mathcal{M}$ fixes what elements we are interested in, and what we mean by each term

# Models

## Meaning

A model $\mathcal{M}$ fixes what elements we are interested in, and what we mean by each term

## Fix universe

For a particular $\mathcal{M}$, the universe $U^{\mathcal{M}}$ contains all elements that we are interested in.

# Models

## Meaning

A model $\mathcal{M}$ fixes what elements we are interested in, and what we mean by each term

## Fix universe

For a particular $\mathcal{M}$, the universe $U^{\mathcal{M}}$ contains all elements that we are interested in.

## Meaning of terms

For a particular $\mathcal{M}$ and a particular term $t$, the meaning of $t$ in $\mathcal{M}$, denoted $t^{\mathcal{M}}$, is a particular subset of $U^{\mathcal{M}}$.

# Example 1A

For our examples, we have
$\texttt{Term} = \{\texttt{cats}, \texttt{humans}, \texttt{Greeks}, \ldots\}$.

# Example 1A

For our examples, we have
$\texttt{Term} = \{\texttt{cats}, \texttt{humans}, \texttt{Greeks}, \ldots\}$.

## First meaning $\mathcal{M}$

- $U^{\mathcal{M}}$: the set of all living beings,

# Example 1A

For our examples, we have
$\texttt{Term} = \{\texttt{cats}, \texttt{humans}, \texttt{Greeks}, \ldots\}$.

**First meaning $\mathcal{M}$**

- $U^{\mathcal{M}}$: the set of all living beings,
- $\texttt{cat}^{\mathcal{M}}$ the set of all cats,

# Example 1A

For our examples, we have
$\mathtt{Term} = \{\mathtt{cats}, \mathtt{humans}, \mathtt{Greeks}, \ldots\}$.

## First meaning $\mathcal{M}$

- $U^{\mathcal{M}}$: the set of all living beings,
- $\mathtt{cat}^{\mathcal{M}}$ the set of all cats,
- $\mathtt{humans}^{\mathcal{M}}$ the set of all humans,
- ...

# Example 1B

Consider the same $\texttt{Term} = \{\texttt{cats}, \texttt{humans}, \texttt{Greeks}, \ldots\}$.

# Example 1B

Consider the same $\texttt{Term} = \{\texttt{cats}, \texttt{humans}, \texttt{Greeks}, \ldots\}$.

## Second meaning $\mathcal{M}'$

- $U^{\mathcal{M}'}$: A set of 100 playing cards, *depicting* living beings,

# Example 1B

Consider the same $\mathtt{Term} = \{\mathtt{cats}, \mathtt{humans}, \mathtt{Greeks}, \ldots\}$.

## Second meaning $\mathcal{M}'$

- $U^{\mathcal{M}'}$: A set of 100 playing cards, *depicting* living beings,
- $\mathtt{cat}^{\mathcal{M}'}$: all cards that show cats,

# Example 1B

Consider the same $\texttt{Term} = \{\texttt{cats}, \texttt{humans}, \texttt{Greeks}, \ldots\}$.

**Second meaning $\mathcal{M}'$**

- $U^{\mathcal{M}'}$: A set of 100 playing cards, *depicting* living beings,
- $\texttt{cat}^{\mathcal{M}'}$: all cards that show cats,
- $\texttt{humans}^{\mathcal{M}'}$: all cards that show humans,
- $\ldots$

# Example 2A

Consider the following set of terms:

$\texttt{Term} = \{\texttt{even}, \texttt{odd}, \texttt{belowfour}\}$

# Example 2A

Consider the following set of terms:

$\texttt{Term} = \{\texttt{even}, \texttt{odd}, \texttt{belowfour}\}$

## First meaning $\mathcal{M}_1$

- $U^{\mathcal{M}_1} = \mathbb{N}$,

# Example 2A

Consider the following set of terms:

$\mathtt{Term} = \{\mathtt{even}, \mathtt{odd}, \mathtt{belowfour}\}$

## First meaning $\mathcal{M}_1$

- $U^{\mathcal{M}_1} = \mathbb{N}$,
- $\mathtt{even}^{\mathcal{M}_1} = \{0, 2, 4, \ldots\}$,

# Example 2A

Consider the following set of terms:

$\mathtt{Term} = \{\mathtt{even}, \mathtt{odd}, \mathtt{belowfour}\}$

## First meaning $\mathcal{M}_1$

- $U^{\mathcal{M}_1} = \mathbb{N}$,
- $\mathtt{even}^{\mathcal{M}_1} = \{0, 2, 4, \ldots\}$,
- $\mathtt{odd}^{\mathcal{M}_1} = \{1, 3, 5, \ldots\}$, and

# Example 2A

Consider the following set of terms:

$\mathtt{Term} = \{\mathtt{even}, \mathtt{odd}, \mathtt{belowfour}\}$

## First meaning $\mathcal{M}_1$

- $U^{\mathcal{M}_1} = \mathbb{N}$,
- $\mathtt{even}^{\mathcal{M}_1} = \{0, 2, 4, \ldots\}$,
- $\mathtt{odd}^{\mathcal{M}_1} = \{1, 3, 5, \ldots\}$, and
- $\mathtt{belowfour}^{\mathcal{M}_1} = \{0, 1, 2, 3\}$.

# Example 2B

Consider the same $\texttt{Term} = \{\texttt{even}, \texttt{odd}, \texttt{belowfour}\}$

# Example 2B

Consider the same $\texttt{Term} = \{\texttt{even}, \texttt{odd}, \texttt{belowfour}\}$

## Second meaning $\mathcal{M}_2$

- $U^{\mathcal{M}_2} = \{a, b, c, \ldots, z\}$,

# Example 2B

Consider the same $\texttt{Term} = \{\texttt{even}, \texttt{odd}, \texttt{belowfour}\}$

## Second meaning $\mathcal{M}_2$

- $U^{\mathcal{M}_2} = \{a, b, c, \ldots, z\}$,
- $\texttt{even}^{\mathcal{M}_2} = \{a, e, i, o, u\}$,

# Example 2B

Consider the same $\mathtt{Term} = \{\mathtt{even}, \mathtt{odd}, \mathtt{belowfour}\}$

## Second meaning $\mathcal{M}_2$

- $U^{\mathcal{M}_2} = \{a, b, c, \ldots, z\}$,
- $\mathtt{even}^{\mathcal{M}_2} = \{a, e, i, o, u\}$,
- $\mathtt{odd}^{\mathcal{M}_2} = \{b, c, d, \ldots\}$, and

# Example 2B

Consider the same $\mathtt{Term} = \{\mathtt{even}, \mathtt{odd}, \mathtt{belowfour}\}$

## Second meaning $\mathcal{M}_2$

- $U^{\mathcal{M}_2} = \{a, b, c, \ldots, z\}$,
- $\mathtt{even}^{\mathcal{M}_2} = \{a, e, i, o, u\}$,
- $\mathtt{odd}^{\mathcal{M}_2} = \{b, c, d, \ldots\}$, and
- $\mathtt{belowfour}^{\mathcal{M}_2} = \emptyset$.

1 Categorical Terms and their Meaning

2 Propositions, Axioms, Lemmas, Proofs
- Categorical Propositions
- Semantics of Propositions
- Axioms, Lemmas and Proofs

3 Manipulating Terms and Propositions

4 Arguments and Syllogisms

# Categorical Propositions

*`All cats are predators`*

expresses a relationship between the terms `cats` (subject) and `predators` (object).

# Categorical Propositions

`All cats are predators`

expresses a relationship between the terms `cats` (subject) and `predators` (object).

**Intended *meaning***

Every thing that is included in the class represented by `cats` is also included in the class represented by `predators`.

# Four Kinds of Categorical Propositions

| | | Quantity | |
|---|---|---|---|
| | | universal | particular |
| Quality | affirmative | `All` $t_1$ `are` $t_2$ | `Some` $t_1$ `are` $t_2$ |
| | negative | `No` $t_1$ `are` $t_2$ | `Some` $t_1$ `are not` $t_2$ |

# Four Kinds of Categorical Propositions

| | | Quantity | |
|---|---|---|---|
| | | universal | particular |
| Quality | affirmative | `All` $t_1$ `are` $t_2$ | `Some` $t_1$ `are` $t_2$ |
| | negative | `No` $t_1$ `are` $t_2$ | `Some` $t_1$ `are not` $t_2$ |

## Example

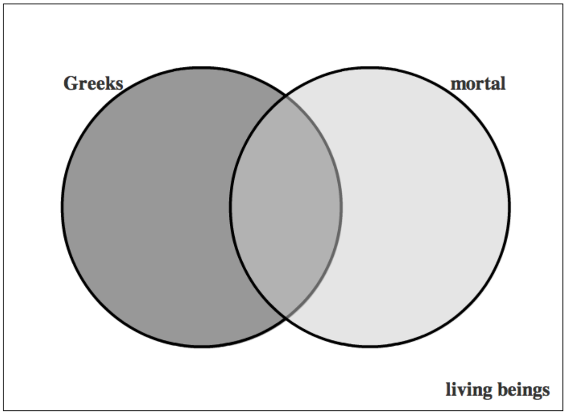
`Some cats are not brave` is a *particular*, *negative* proposition.

# Meaning of Universal Affirmative Propositions

In a particular model $\mathcal{M}$, `All Greeks are mortal` means that $\texttt{Greeks}^{\mathcal{M}}$ is a subset of $\texttt{mortal}^{\mathcal{M}}$

# Meaning of Universal Affirmative Propositions

In a particular model $\mathcal{M}$, `All Greeks are mortal` means that $\texttt{Greeks}^{\mathcal{M}}$ is a subset of $\texttt{mortal}^{\mathcal{M}}$



# More formally...

$$(\texttt{All}\ \textit{subject}\ \texttt{are}\ \textit{object})^{\mathcal{M}} = \begin{cases} T & \text{if } \textit{subject}^{\mathcal{M}} \subseteq \textit{object}^{\mathcal{M}}, \\ F & \text{otherwise} \end{cases}$$

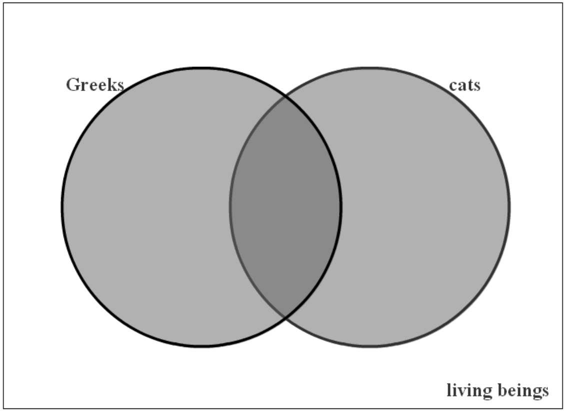
Here $T$ and $F$ represent the logical truth values *true* and *false*, respectively.

# Meaning of Universal Negative Propositions

In a particular model $\mathcal{M}$, `No Greeks are cats` means that the intersection of $\texttt{Greeks}^{\mathcal{M}}$ and of $\texttt{cats}^{\mathcal{M}}$ is empty.

# Meaning of Universal Negative Propositions

In a particular model $\mathcal{M}$, `No Greeks are cats` means that the intersection of $\texttt{Greeks}^{\mathcal{M}}$ and of $\texttt{cats}^{\mathcal{M}}$ is empty.



# More formally...

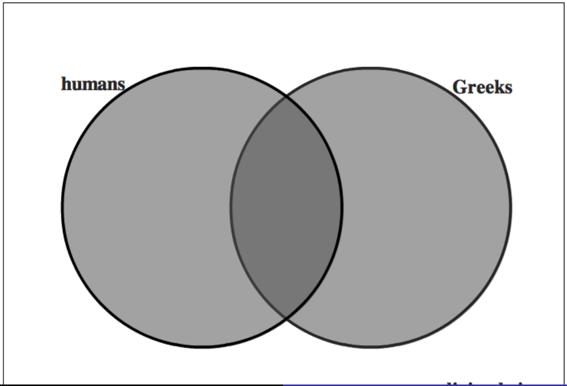
$$(\texttt{No}\ \textit{subject}\ \texttt{are}\ \textit{object})^{\mathcal{M}} = \begin{cases} T & \text{if } \textit{subject}^{\mathcal{M}} \cap \textit{object}^{\mathcal{M}} = \emptyset, \\ F & \text{otherwise} \end{cases}$$

# Meaning of Particular Affirmative Propositions

In a particular model $\mathcal{M}$, `Some humans are Greeks` means that the intersection of $\texttt{humans}^{\mathcal{M}}$ and of $\texttt{Greeks}^{\mathcal{M}}$ is not empty.

# Meaning of Particular Affirmative Propositions

In a particular model $\mathcal{M}$, `Some humans are Greeks` means that the intersection of $\texttt{humans}^{\mathcal{M}}$ and of $\texttt{Greeks}^{\mathcal{M}}$ is not empty.



# More formally...

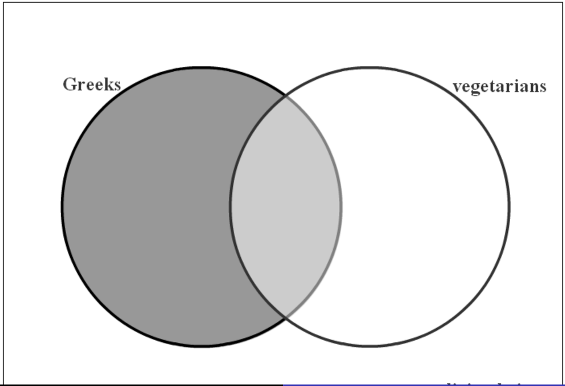
$$(\texttt{Some}\ \mathit{subject}\ \texttt{are}\ \mathit{object})^{\mathcal{M}} = \begin{cases} T & \text{if } \mathit{subject}^{\mathcal{M}} \cap \mathit{object}^{\mathcal{M}} \neq \emptyset, \\ F & \text{otherwise} \end{cases}$$

# Meaning of Particular Negative Propositions

In a particular model $\mathcal{M}$, `Some Greeks are not vegetarians` means that the difference of $\texttt{Greeks}^{\mathcal{M}}$ and $\texttt{vegetarians}^{\mathcal{M}}$ is not empty.

# Meaning of Particular Negative Propositions

In a particular model $\mathcal{M}$, `Some Greeks are not vegetarians` means that the difference of `Greeks`$^{\mathcal{M}}$ and `vegetarians`$^{\mathcal{M}}$ is not empty.



# More formally...

$$(\texttt{Some}\ \textit{subject}\ \texttt{are not}\ \textit{object})^{\mathcal{M}} = \begin{cases} T & \text{if } \textit{subject}^{\mathcal{M}} / \textit{object}^{\mathcal{M}} \neq \emptyset, \\ F & \text{otherwise} \end{cases}$$

# Axioms

*Axioms* are propositions that are assumed to hold.

# Axioms

*Axioms* are propositions that are assumed to hold.

**Axiom (HM)**

*The proposition* `All humans are mortal` *holds.*

# Axioms

*Axioms* are propositions that are assumed to hold.

**Axiom (HM)**

*The proposition* `All humans are mortal` *holds.*

**Axiom (GH)**

*The proposition* `All Greeks are humans` *holds.*

# Graphical Notation

$$\frac{}{\texttt{All humans are mortal}}[\text{HumansMortality}]$$

# Lemmas

Lemmas are affirmations that follow from all known facts.

# Lemmas

Lemmas are affirmations that follow from all known facts.

**Proof obligation**

A lemma must be followed by a proof that demonstrates how it follows from known facts.

# Trivial Example of Proof

**Lemma**

*The proposition* `All humans are mortal` *holds.*

# Trivial Example of Proof

**Lemma**

*The proposition* `All humans are mortal` *holds.*

**Proof.**

$$\frac{}{\texttt{All humans are mortal}}\,[\mathrm{HM}]$$

□

# Unusual Models

We can choose any model for our terms, also “unusual” ones.

# Unusual Models

We can choose any model for our terms, also “unusual” ones.

**Example**

$U^{\mathcal{M}} = \{0, 1\}$, $\texttt{humans}^{\mathcal{M}} = \{0\}$, $\texttt{mortal}^{\mathcal{M}} = \{1\}$

# Unusual Models

We can choose any model for our terms, also "unusual" ones.

**Example**

$U^{\mathcal{M}} = \{0, 1\}$, $\texttt{humans}^{\mathcal{M}} = \{0\}$, $\texttt{mortal}^{\mathcal{M}} = \{1\}$

Here

`All humans are mortal`

does not hold.

# Asserting Axioms

## Purpose of axioms

By asserting an axiom $A$, we are focusing our attention to only those models $\mathcal{M}$ for which $A^{\mathcal{M}} = T$.

# Asserting Axioms

## Purpose of axioms

By asserting an axiom $A$, we are focusing our attention to only those models $\mathcal{M}$ for which $A^{\mathcal{M}} = T$.

## Consequence

The lemmas that we prove while utilizing an axiom only hold in the models in which the axiom holds.

# Asserting Axioms

## Purpose of axioms

By asserting an axiom $A$, we are focusing our attention to only those models $\mathcal{M}$ for which $A^{\mathcal{M}} = T$.

## Consequence

The lemmas that we prove while utilizing an axiom only hold in the models in which the axiom holds.

## Validity

A proposition is called *valid*, if it holds in all models.

1. Categorical Terms and their Meaning
2. Propositions, Axioms, Lemmas, Proofs
3. Manipulating Terms and Propositions
   - Complement
   - Conversion
   - Contraposition
   - Obversion
   - Combinations
4. Arguments and Syllogisms

# Complement

We allow ourselves to put `non` in front of a term.

# Complement

We allow ourselves to put `non` in front of a term.

**Meaning of complement**

In a model $\mathcal{M}$, the meaning of `non` $t$ is the complement of the meaning of $t$

# Complement

We allow ourselves to put $\mathtt{non}$ in front of a term.

## Meaning of complement

In a model $\mathcal{M}$, the meaning of $\mathtt{non}$ $t$ is the complement of the meaning of $t$

## More formally

In a model $\mathcal{M}$, $(\mathtt{non}\ t)^{\mathcal{M}} = U^{\mathcal{M}} / t^{\mathcal{M}}$

# Double Complement

**Axiom (NonNon)**

*For any term t, the term* non non *t is considered equal to t.*

# Double Complement

**Axiom (NonNon)**

*For any term $t$, the term $\mathtt{non}\ \ \mathtt{non}\ t$ is considered equal to $t$.*

$$\frac{\cdots t \cdots}{\cdots \mathtt{non}\ \ \mathtt{non}\ t \cdots}[\text{NNI}]$$

$$\frac{\cdots \mathtt{non}\ \ \mathtt{non}\ t \cdots}{\cdots t \cdots}[\text{NNE}]$$

# Rule Schema

$$\frac{\cdots t \cdots}{\cdots \texttt{non non } t \cdots}\,[\mathrm{NNI}]$$

is a rule schema. An instance is:

$$\frac{\texttt{Some } t_1 \texttt{ are } t_2}{\texttt{Some non non } t_1 \texttt{ are } t_2}$$

# Definitions

We allow ourselves to state definitions that may be convenient. Definitions are similar to axioms; they fix the properties of a particular item for the purpose of a discussion.

# Definitions

We allow ourselves to state definitions that may be convenient. Definitions are similar to axioms; they fix the properties of a particular item for the purpose of a discussion.

**Definition (ImmDef)**

The term `immortal` is considered equal to the term `non mortal`.

# Writing a Proof Graphically

**Lemma**

*The proposition* `All humans are non immortal` *holds.*

# Writing a Proof Graphically

**Lemma**

*The proposition* `All humans are non immortal` *holds.*

**Proof.**

$$\dfrac{\dfrac{\dfrac{}{\texttt{All humans are mortal}}\,[\mathrm{HM}]}{\texttt{All humans are non non mortal}}\,[\mathrm{NNI}]}{\texttt{All humans are non immortal}}\,[\mathrm{ImmDef}]$$

# Writing a Text-based Proof

**Lemma**

*The proposition* `All humans are non immortal` *holds.*

# Writing a Text-based Proof

**Lemma**

*The proposition* `All humans are non immortal` *holds.*

**Proof.**

| | | |
|---|---|---|
| 1 | `All humans are mortal` | HM |
| 2 | `All humans are non non mortal` | NNI 1 |
| 3 | `All humans are non immortal` | ImmDef 2 |

□

# Conversion switches subject and object

**Definition (ConvDef)**

For all terms $t_1$ and $t_2$, we define

$$\begin{aligned}
\texttt{convert}(\texttt{All } t_1 \texttt{ are } t_2) &= \texttt{All } t_2 \texttt{ are } t_1 \\
\texttt{convert}(\texttt{Some } t_1 \texttt{ are } t_2) &= \texttt{Some } t_2 \texttt{ are } t_1 \\
\texttt{convert}(\texttt{No } t_1 \texttt{ are } t_2) &= \texttt{No } t_2 \texttt{ are } t_1 \\
\texttt{convert}(\texttt{Some } t_1 \texttt{ are not } t_2) &= \texttt{Some } t_2 \texttt{ are not } t_1
\end{aligned}$$

# Which Conversions Hold?

If

`All Greeks are humans`

holds in a model, then does

`All humans are Greeks`

hold?

# Valid Conversions

**Axiom (ConvE1)**

*If, for some terms $t_1$ and $t_2$, the proposition*

$$\texttt{convert}(\texttt{Some}\ t_1\ \texttt{are}\ t_2)$$

*holds, then the proposition*

$$\texttt{Some}\ t_1\ \texttt{are}\ t_2$$

*also holds.*

# Valid Conversions

## Axiom (ConvE2)

*If, for some terms $t_1$ and $t_2$, the proposition*

$$\texttt{convert}(\texttt{No}\ t_1\ \texttt{are}\ t_2)$$

*holds, then the proposition*

$$\texttt{No}\ t_1\ \texttt{are}\ t_2$$

*also holds.*

# In Graphical Notation

In graphical notation, two rules correspond to the two cases.

$$\frac{convert(\texttt{Some } t_1 \texttt{ are } t_2)}{\texttt{Some } t_1 \texttt{ are } t_2}\,[\mathrm{ConvE}_1]$$

$$\frac{convert(\texttt{No } t_1 \texttt{ are } t_2)}{\texttt{No } t_1 \texttt{ are } t_2}\,[\mathrm{ConvE}_2]$$

# Example

**Axiom (AC)**

*The proposition* `Some animals are cats` *holds.*

# Example

**Axiom (AC)**

*The proposition* `Some animals are cats` *holds.*

**Lemma**

*The proposition* `Some cats are animals` *holds.*

# Proof

$$\dfrac{\dfrac{\dfrac{}{\texttt{Some animals are cats}}\,[\mathrm{AC}]}{\texttt{convert(Some cats are animals)}}\,[\mathrm{ConvDef}]}{\texttt{Some cats are animals}}\,[\mathrm{ConvE}_1]$$

# Example (text-based proof)

**Proof.**

| | | |
|---|---|---|
| 1 | `Some animals are cats` | AC |
| 2 | `convert(Some cats are animals)` | ConvDef 1 |
| 3 | `Some cats are animals` | $\text{ConvE}_1$ 2 |

□

# Contraposition switches and complements

## Definition (ContrDef)

For all terms $t_1$ and $t_2$, we define

$$\begin{aligned}
& \texttt{contrapose}(\texttt{All}\ t_1\ \texttt{are}\ t_2) \\
= \ & \texttt{All non}\ t_2\ \texttt{are non}\ t_1 \\
& \texttt{contrapose}(\texttt{Some}\ t_1\ \texttt{are}\ t_2) \\
= \ & \texttt{Some non}\ t_2\ \texttt{are non}\ t_1 \\
& \texttt{contrapose}(\texttt{No}\ t_1\ \texttt{are}\ t_2) \\
= \ & \texttt{No non}\ t_2\ \texttt{are non}\ t_1 \\
& \texttt{contrapose}(\texttt{Some}\ t_1\ \texttt{are not}\ t_2) \\
= \ & \texttt{Some non}\ t_2\ \texttt{are not non}\ t_1
\end{aligned}$$

# For which propositions is contraposition valid?

$$\frac{\mathit{contrapose}(\texttt{All } t_1 \texttt{ are } t_2)}{\texttt{All } t_1 \texttt{ are } t_2}\,[\mathrm{ContrE}_1]$$

# For which propositions is contraposition valid?

$$\frac{\mathit{contrapose}(\texttt{All } t_1 \texttt{ are } t_2)}{\texttt{All } t_1 \texttt{ are } t_2}\ [\mathrm{ContrE}_1]$$

$$\frac{\mathit{contrapose}(\texttt{Some } t_1 \texttt{ are not } t_2)}{\texttt{Some } t_1 \texttt{ are not } t_2}\ [\mathrm{ContrE}_2]$$

# Obversion switches quality and complements object

## Definition (ObvDef)

For all terms $t_1$ and $t_2$, we define

$$\begin{aligned}
\texttt{obvert}(\texttt{All } t_1 \texttt{ are } t_2) &= \texttt{No } t_1 \texttt{ are non } t_2 \\
\texttt{obvert}(\texttt{Some } t_1 \texttt{ are } t_2) &= \texttt{Some } t_1 \texttt{ are not non } t_2 \\
\texttt{obvert}(\texttt{No } t_1 \texttt{ are } t_2) &= \texttt{All } t_1 \texttt{ are non } t_2 \\
\texttt{obvert}(\texttt{Some } t_1 \texttt{ are not } t_2) &= \texttt{Some } t_1 \texttt{ are non } t_2
\end{aligned}$$

# Examples

Obversion switches quality and complements object

# Examples

Obversion switches quality and complements object

## Example 1

```
obvert(All Greeks are humans)
 = No Greeks are non humans
```

# Examples

Obversion switches quality and complements object

## Example 1

```
obvert(All Greeks are humans)
 = No Greeks are non humans
```

## Example 2

```
obvert(Some animals are cats)
 = Some animals are not non cats
```

# Validity of Obversion

Obversion is valid for all kinds of propositions.

**Axiom (ObvE)**

*If, for some proposition p*

$$\texttt{obvert}(p)$$

*holds, then the proposition p also holds.*

# Validity of Obversion

Obversion is valid for all kinds of propositions.

**Axiom (ObvE)**

*If, for some proposition p*

$$\texttt{obvert}(p)$$

*holds, then the proposition p also holds.*

$$\frac{obvert(p)}{p}\,[\mathrm{ObvE}]$$

# Example

**Axiom (SHV)**

*The proposition* `Some humans are vegans` *holds.*

# Example

**Axiom (SHV)**

*The proposition* `Some humans are vegans` *holds.*

**Lemma (NNVeg)**

*The proposition* `Some humans are not non vegans` *holds.*

# Proof

$$\frac{}{\texttt{Some humans are vegans}}\,[\text{SHV}]$$

$$\frac{}{\texttt{Some humans are non non vegans}}\,[\text{NNI}]$$

$$\frac{}{\texttt{obvert(Some humans are not non vegans)}}\,[\text{ObvDef}]$$

$$\frac{}{\texttt{Some humans are not non vegans}}\,[\text{C}$$

# Proof (text-based)

**Proof.**

| | | |
|---|---|---|
| 1 | `Some humans are vegans` | SHV |
| 2 | `Some humans are non non vegans` | NNI 1 |
| 3 | `obvert(Some humans are not non vegans)` | ObvDef 2 |
| 4 | `Some humans are not non vegans` | ObvE 3 |

□

# Another Lemma

**Lemma (SomeNon)**

*For all terms $t_1$ and $t_2$, if the proposition* `Some non` $t_1$ `are non` $t_2$ *holds, then the proposition* `Some non` $t_2$ `are not` $t_1$ *also holds.*

# Another Lemma

**Lemma (SomeNon)**

*For all terms $t_1$ and $t_2$, if the proposition* `Some non` $t_1$ `are non` $t_2$ *holds, then the proposition* `Some non` $t_2$ `are not` $t_1$ *also holds.*

A lemma of the form "If $p_1$ then $p_2$" is valid, if in every model in which the proposition $p_1$ holds, the proposition $p_2$ also holds.

# Proof

## Lemma (SomeNon)

*For all terms $t_1$ and $t_2$, if the proposition* `Some non` $t_1$ `are non` $t_2$ *holds, then the proposition* `Some non` $t_2$ `are not` $t_1$ *also holds.*

# Proof

## Lemma (SomeNon)

*For all terms $t_1$ and $t_2$, if the proposition* `Some non` $t_1$ `are non` $t_2$ *holds, then the proposition* `Some non` $t_2$ `are not` $t_1$ *also holds.*

## Proof.

| | | |
|---|---|---|
| 1 | `Some non` $t_1$ `are non` $t_2$ | premise |
| 2 | `convert(Some non` $t_2$ `are non` $t_1$`)` | ConvDef 1 |
| 3 | `Some non` $t_2$ `are non` $t_1$ | $\text{ConvE}_1$ 2 |
| 4 | `obvert(Some non` $t_2$ `are not` $t_1$`)` | ObvDef 3 |
| 5 | `Some non` $t_2$ `are not` $t_1$ | ObvE 4 |

□

# "iff" means "if and only if"

**Lemma (AllNonNon)**

*For any terms $t_1$ and $t_2$, the proposition* `All non` $t_1$ `are non` $t_2$ *holds iff the proposition* `All` $t_2$ `are` $t_1$ *holds.*

# “iff” means “if and only if”

**Lemma (AllNonNon)**

*For any terms $t_1$ and $t_2$, the proposition* `All non` $t_1$ `are non` $t_2$ *holds iff the proposition* `All` $t_2$ `are` $t_1$ *holds.*

$$\frac{\texttt{All non } t_1 \texttt{ are non } t_2}{\texttt{All } t_2 \texttt{ are } t_1}$$

$$\frac{\texttt{All } t_2 \texttt{ are } t_1}{\texttt{All non } t_1 \texttt{ are non } t_2}$$

1. Categorical Terms and their Meaning
2. Propositions, Axioms, Lemmas, Proofs
3. Manipulating Terms and Propositions
4. Arguments and Syllogisms
   - Arguments
   - Syllogisms
   - Barbara
   - Fun With Barbara

# Argument

An argument has the form

If *premises* then *conclusion*

# Argument

An argument has the form

If *premises* then *conclusion*

Sometimes also

*premises* therefore *conclusion*

# Argument

An argument has the form

If *premises* then *conclusion*

Sometimes also

*premises* therefore *conclusion*

Example:

**Lemma (SomeNon)**

*For all terms $t_1$ and $t_2$, if the proposition* `Some non` $t_1$ `are non` $t_2$ *holds, then the proposition* `Some non` $t_2$ `are not` $t_1$ *also holds.*

# Syllogisms

A syllogism is an argument with two premises, in which three different terms occur, and in which every term occurs twice, but never twice in the same proposition.

## Example

*All cats are predators.*
*Some animals are cats.*
*Therefore, all animals are predators.*

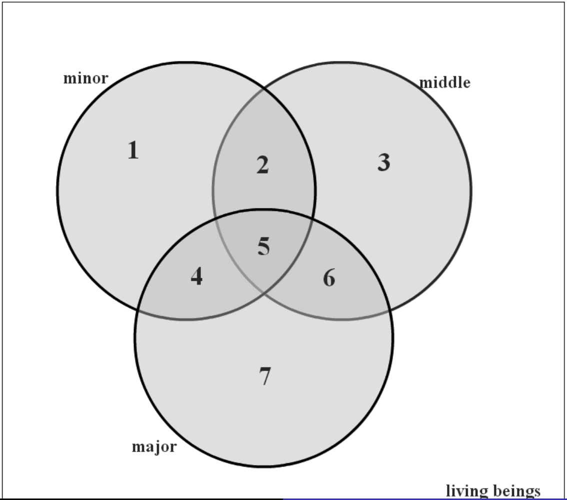
# Barbara

**Axiom (B)**

*For all terms minor, middle, and major, if* `All` *middle* `are` *major holds, and* `All` *minor* `are` *middle holds, then* `All` *minor* `are` *major also holds.*

$$\frac{\texttt{All}\ \textit{middle}\ \texttt{are}\ \textit{major} \qquad \texttt{All}\ \textit{minor}\ \texttt{are}\ \textit{middle}}{\texttt{All}\ \textit{minor}\ \texttt{are}\ \textit{major}}\ [B]$$

# Why is Barbara valid?



# Example

**Lemma**

*The proposition* `All Greeks are mortal` *holds.*

# Example

**Lemma**

*The proposition* `All Greeks are mortal` *holds.*

**Proof.**

| | | |
|---|---|---|
| 1 | `All Greeks are humans` | GH |
| 2 | `All humans are mortal` | HM |
| 3 | `All Greeks are mortal` | B 1,2 |

□

# Officers as Poultry?

## Premises

- No ducks waltz.
- No officers ever decline to waltz.
- All my poultry are ducks.

# Officers as Poultry?

## Premises

- No ducks waltz.
- No officers ever decline to waltz.
- All my poultry are ducks.

## Conclusion

No officers are my poultry.

# Formulation in Term Logic

**Lemma (No-Officers-Are-My-Poutry)**

*If*

- `No ducks are things-that-waltz` *holds,*
- `No officers are non things-that-waltz` *holds, and*
- `All my-poutry are ducks` *holds,*

*then* `No officers are my-poultry` *also holds.*

# Proof

| | | |
|---|---|---|
| 1 | `No officers are non things-that-waltz` | premise |
| 2 | `obvert(All officers are things-that-waltz)` | ObvDef 1 |
| 3 | `All officers are things-that-waltz)` | ObvE 2 |
| 4 | `No ducks are things-that-waltz)` | premise |
| 5 | `convert(No things-that-waltz are ducks)` | ConvDef 4 |
| 6 | `No things-that-waltz are ducks` | $\text{ConvE}_2$ 5 |

# Proof (continued)

| | | |
|---|---|---|
| 7 | `No things-that-waltz are non non ducks` | NNI 6 |
| 8 | `obvert(All things-that-waltz are non ducks)` | ObvDef 7 |
| 9 | `All things-that-waltz are non ducks` | ObvE 8 |
| 10 | `All my-poultry are ducks` | premise |
| 11 | `All my-poultry are non non ducks` | NNI 10 |
| 12 | `All non non my-poultry are non non ducks` | NNI 11 |

# Proof (continued)

| | | |
|---|---|---|
| 13 | `contrapose(All non ducks are non my-poultry)` | ContrDef 12 |
| 14 | `All non ducks are non my-poultry` | $\text{ContrE}_1$ 13 |
| 15 | `All things-that-waltz are non my-poultry` | B 9,14 |
| 16 | `All officers are non my-poultry` | B 3,15 |
| 17 | `obvert(No officers are my-poultry)` | ObvDef 16 |
| 18 | `No officers are my-poultry` | ObvE 17 |

□

# Admin

- Assignment 1: out on module homepage; due 26/8, 11:00am
- Coq Homework 1: out on module homepage; due 27/8, 9:30pm
- Monday, Wednesday: Office hours
- Tuesday: Tutorials (clarification of assignment)
- Wednesday: Labs (Coq Homework 1; start *earlier!*)