Learning from Observations

Chapter 18

Sections 1-3

Outline

- Learning
- Hypothesis Spaces
- Decision Trees
- **Naïve Bayes**
  - Not in the text
- Training and Testing

What is Learning

- Memorizing something
- Learning facts through observation and exploration
- Generalizing a concept from experience

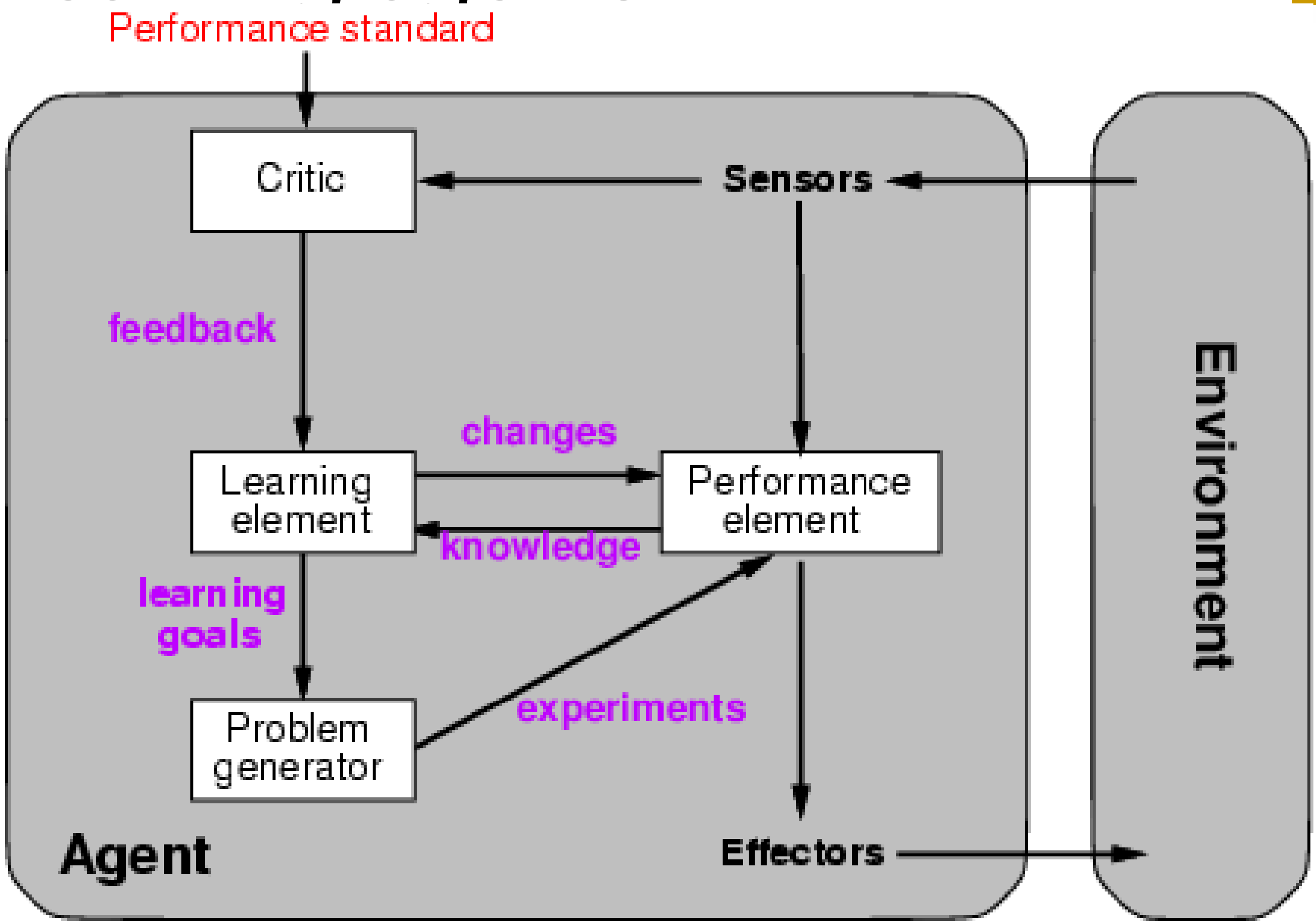
"Learning denotes changes in the system that are **adaptive** in the sense that they enable the system to do the task or tasks drawn from the **same population** more efficiently and more effectively the next time" – Herb Simon

Why is it necessary?

Three reasons:

- Unknown environment – need to deploy an agent in an unfamiliar territory
- Save labor – we may not have the resources to encode knowledge
- Can’t explicitly encode knowledge – may lack the ability to articulate necessary knowledge.

Learning agents



Learning element

- Design of a learning element is affected by
  - Which components of the performance element are to be learned
  - What feedback is available to learn these components
  - What representation is used for the components
- Type of feedback:
  - Supervised learning: correct answers for each example
  - Unsupervised learning: correct answers not given
  - Reinforcement learning: occasional rewards

Induction

- Making predictions about the future based on the past.

  If asked why we believe the sun will rise tomorrow, we shall naturally answer, “Because it has always risen every day.” We have a firm belief that it will rise in the future, because it has risen in the past. – Bertrand Russell

- Is induction sound? Why believe that the future will look similar to the past?

Inductive learning

- Simplest form: learn a function from examples

f is the target function

An example is a pair (x, $f(x)$)

Problem: find a hypothesis h

such that $h \approx f$

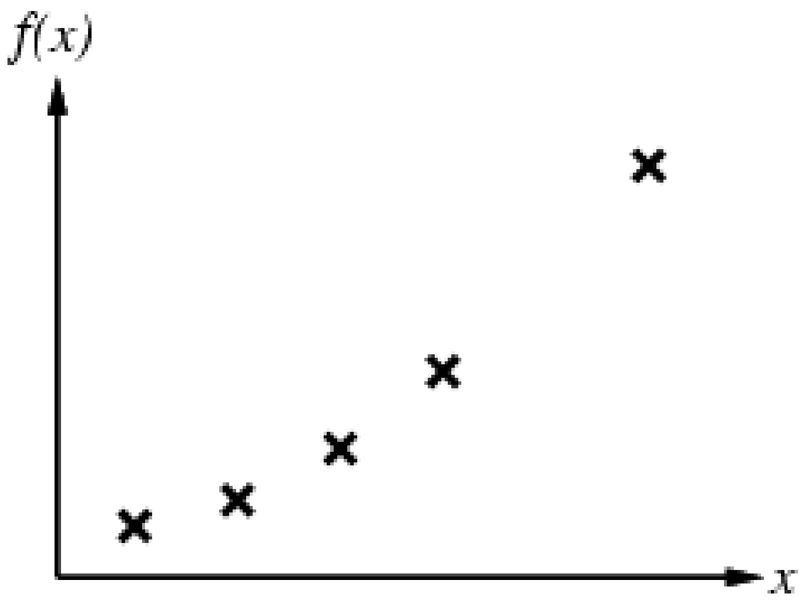
given a training set of examples

This is a highly simplified model of real learning:

- Ignores prior knowledge
- Assumes examples are given

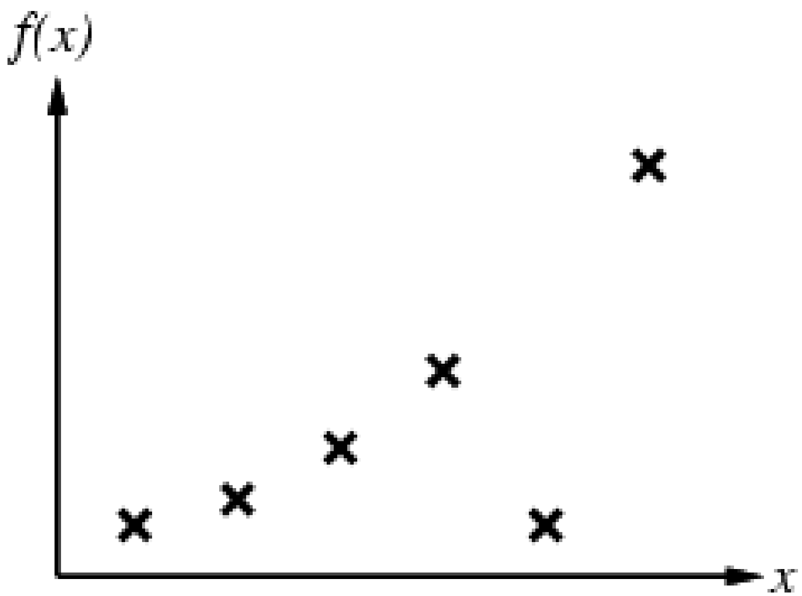
Inductive learning method

- Memorization
- Noise
  - Unreliable function
  - Unreliable sensors



Inductive learning method

- Construct/adjust *h* to agree with *f* on training set
- (*h* is consistent if it agrees with *f* on all examples)
- E.g., curve fitting:



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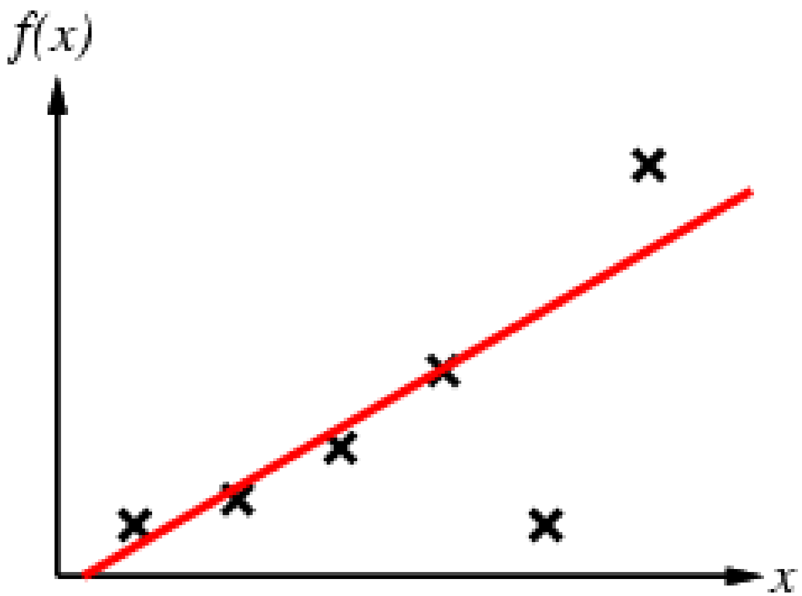

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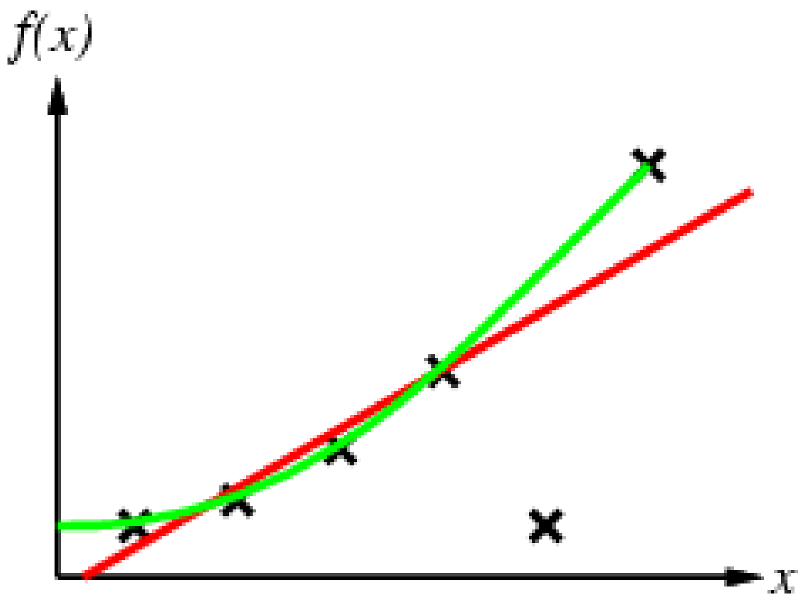

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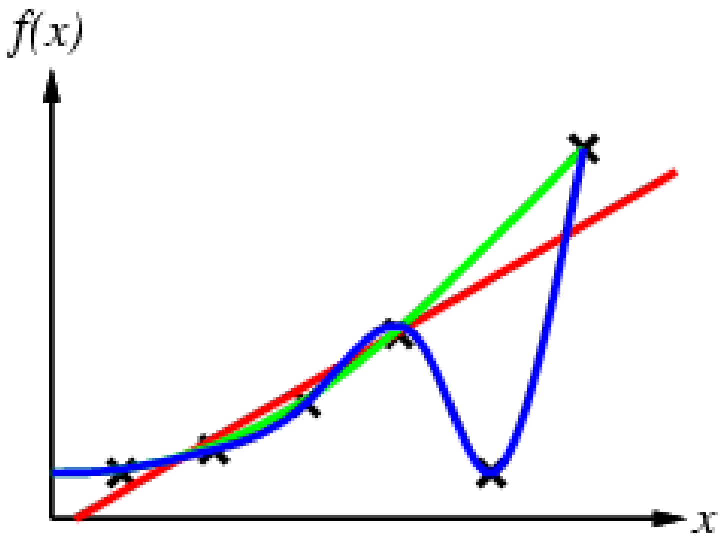

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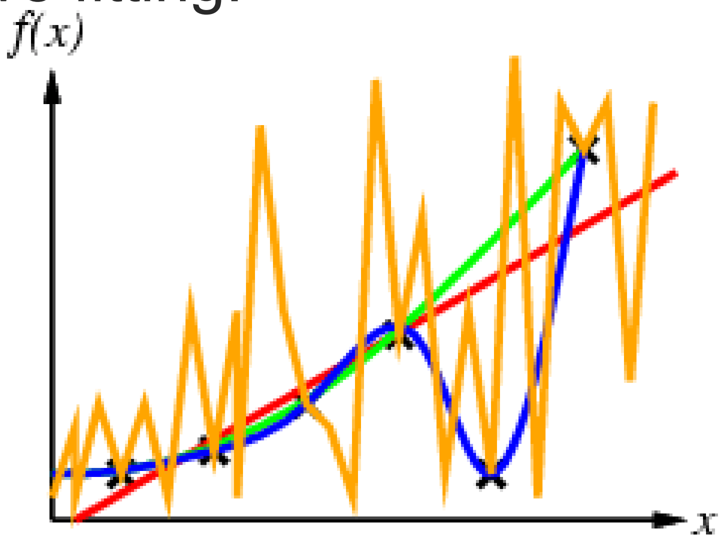

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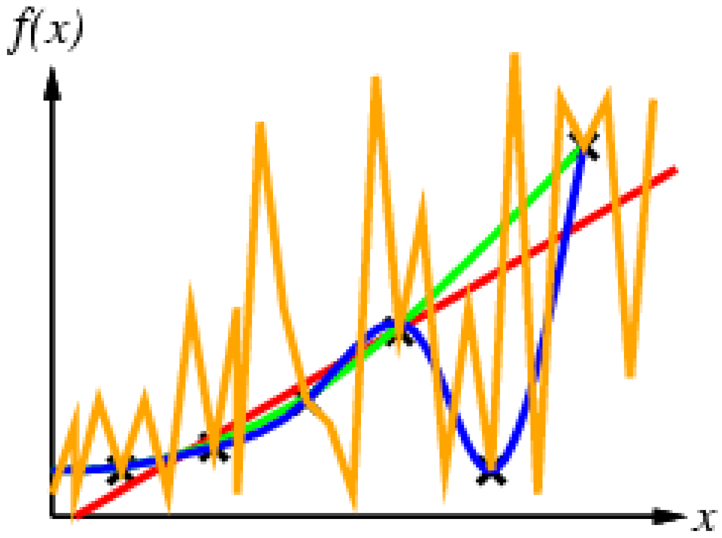
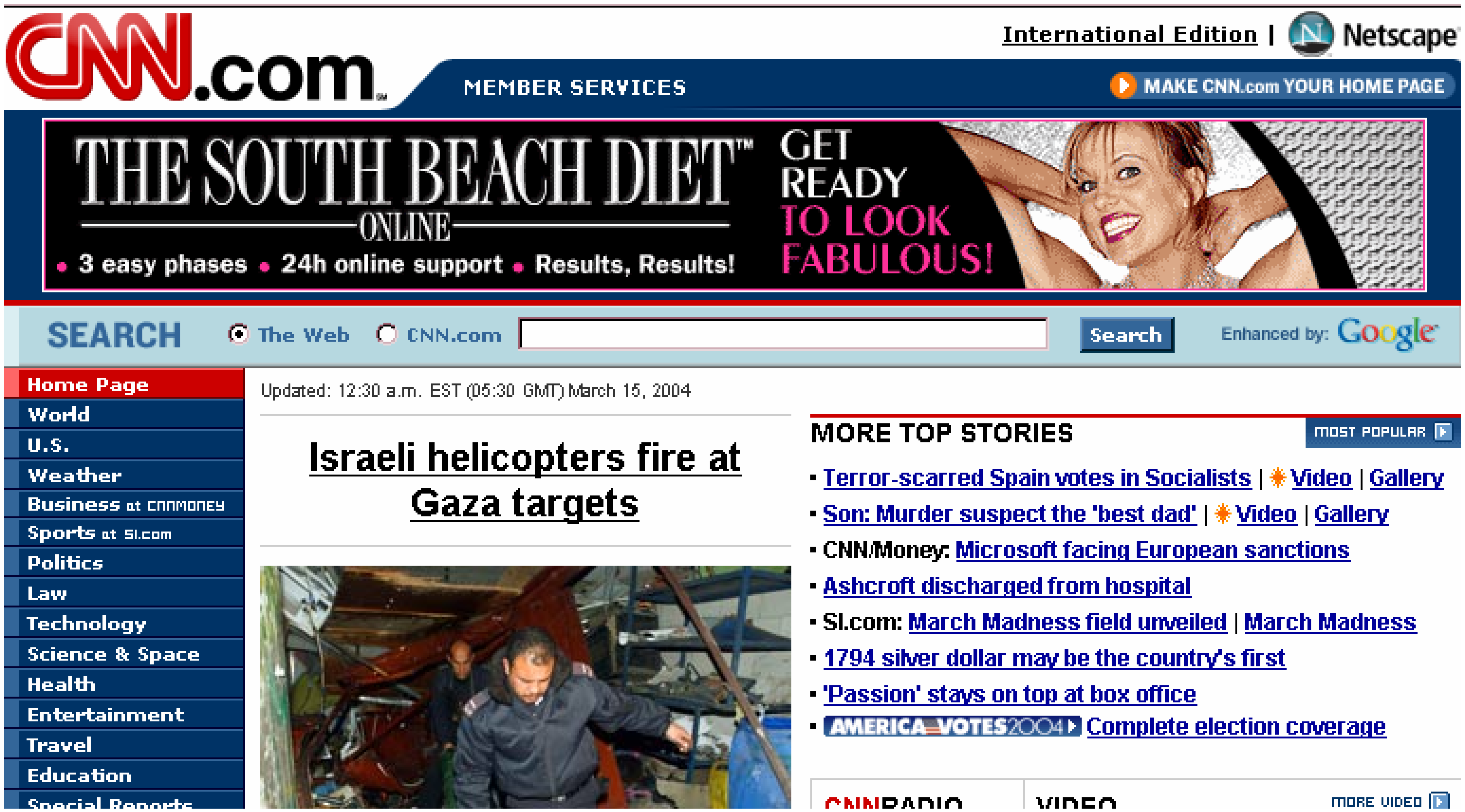


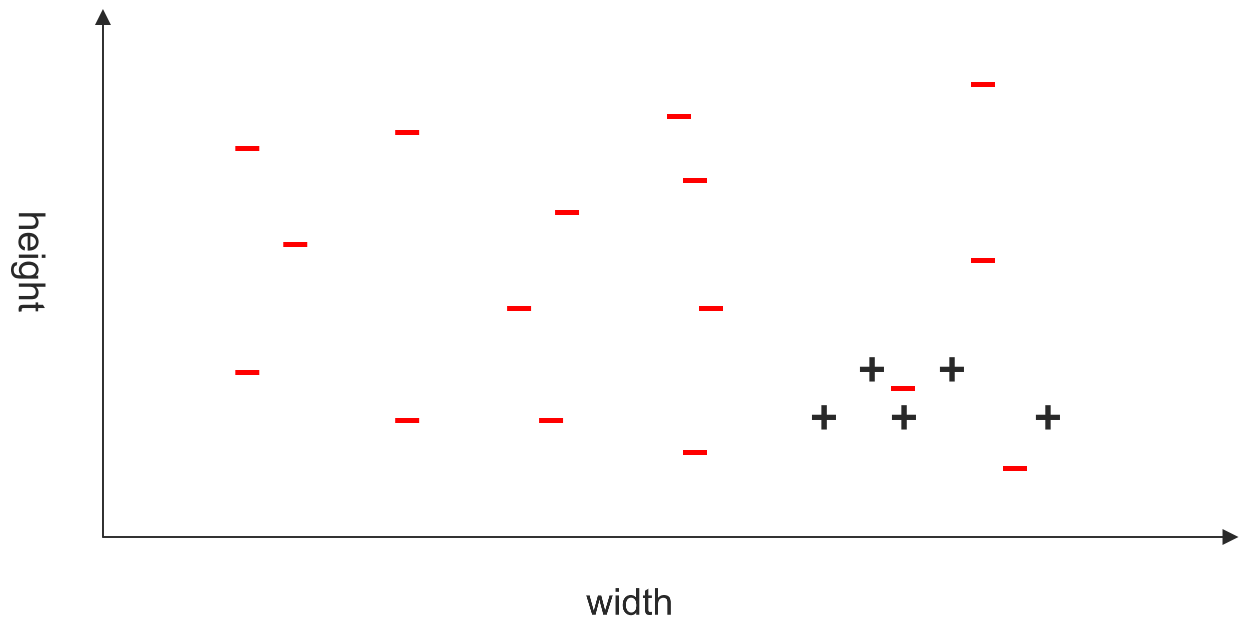
- Ockham's razor: prefer the simplest hypothesis consistent with data

An application: Ad blocking



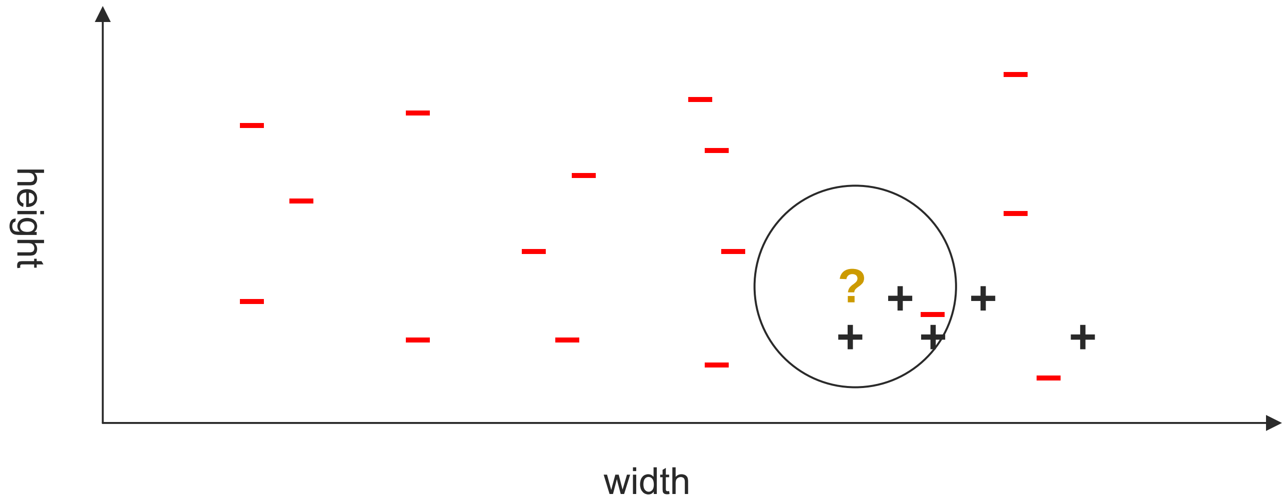
Learning Ad blocking

- Width and height of image
- Binary Classification: Ad or ¬Ad?



Nearest Neighbor

- A type of instance based learning
- Remember all of the past instances
- Use the nearest old data point as answer



- Generalize to kNN, that is take the average class of the closest k neighbors.

Application: Eating out

Problem: Decide on a restaurant, based on the following attributes:

1. Alternate: is there an alternative restaurant nearby?
2. Bar: is there a comfortable bar area to wait in?
3. Fri/Sat: is today Friday or Saturday?
4. Hungry: are we hungry?
5. Patrons: number of people in the restaurant (None, Some, Full)
6. Price: price range ($, $$, $$$)
7. Raining: is it raining outside?
8. Reservation: have we made a reservation?
9. Type: kind of restaurant (French, Italian, Thai, Burger)
10. WaitEstimate: estimated waiting time (0-10, 10-30, 30-60, >60)

Attribute representation

- Examples described by **attribute** or **feature values** (Boolean, discrete, continuous)
- E.g., situations where I will/won't wait for a table:

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

- **Classification** of examples is **positive** (T) or **negative** (F)

Bayes Rule

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

Which is shorthand for:

$$P(Y = y_i|X = x_j) = \frac{P(X = x_j|Y = y_i)P(Y = y_i)}{P(X = x_j)}$$

Naïve Bayes Classifier

- Calculate most probable function value

$$V_{map} = \text{argmax } P(v_j | a_1, a_2, \ldots, a_n)$$

$$= \text{argmax } \frac{P(a_1, a_2, \ldots, a_n | v_j)\, P(v_j)}{P(a_1, a_2, \ldots, a_n)}$$

$$= \text{argmax } P(a_1, a_2, \ldots, a_n | v_j)\, P(v_j)$$

Naïve assumption: $P(a_1, a_2, \ldots, a_n) = P(a_1)P(a_2) \ldots P(a_n)$

Naïve Bayes Algorithm

NaïveBayesLearn(*examples)*

For each target value v_j

$P'(v_j) \leftarrow$ estimate $P(v_j)$

For each attribute value a_i of each attribute a

$P'(a_i|v_j) \leftarrow$ estimate $P(a_i|v_j)$

ClassfyingNewInstance(x)

$$v_{nb} = \operatorname*{argmax}_{v_j \, \varepsilon \, V} P'(v_j) \prod_{a_j \, \varepsilon \, x} P'(a_i|v_j)$$

An Example

(due to MIT's open coursework slides)

f_1	f_2	f_3	f_4	**y**
0	1	1	0	**1**
0	0	1	1	**1**
1	0	1	0	**1**
0	0	1	1	**1**
0	0	0	0	**1**
1	0	0	1	**0**
1	1	0	1	**0**
1	0	0	0	**0**
1	1	0	1	**0**
1	0	1	1	**0**

R_1(1,1) = 1/5: fraction of all positive examples that have feature 1 = 1
R_1(0,1) = 4/5: fraction of all positive examples that have feature 1 = 0

R_1(1,0) = 5/5: fraction of all negative examples that have feature 1 = 1
R_1(0,0) = 0/5: fraction of all negative examples that have feature 1 = 0

Continue calculation of R_2(1,0) …

An Example

(due to MIT's open coursework slides)

f_1	f_2	f_3	f_4	**y**
0	1	1	0	**1**
0	0	1	1	**1**
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1	0	0	1	**0**
1	1	0	1	**0**
1	0	0	0	**0**
1	1	0	1	**0**
1	0	1	1	**0**

(1,1) (0,1) (1,0) (0,0)

$\mathbf{R_1}$ = 1/5, 4/5, 5/5, 0/5

$\mathbf{R_2}$ = 1/5, 4/5, 2/5, 3/5

$\mathbf{R_3}$ = 4/5, 1/5, 1/5, 4/5

$\mathbf{R_4}$ = 2/5, 3/5, 4/5, 1/5

New x = <0, 0, 1, 1>

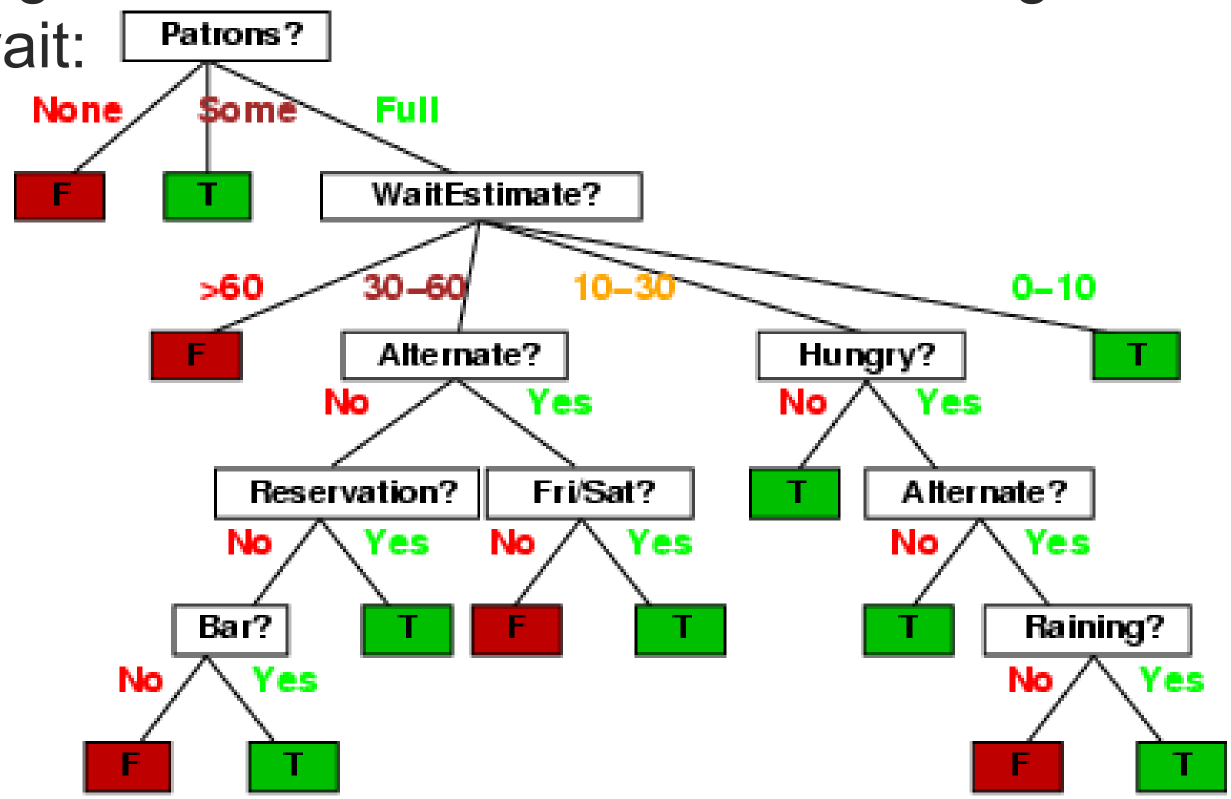
$S(1) = R_1(0,1)*R_2(0,1)*R_3(1,1)*R_4(1,1) = .205$

$S(0) = R_1(0,0)*R_2(0,0)*R_3(1,0)*R_4(1,0) = 0$

S(1) > S(0), so predict v = 1.

Decision trees

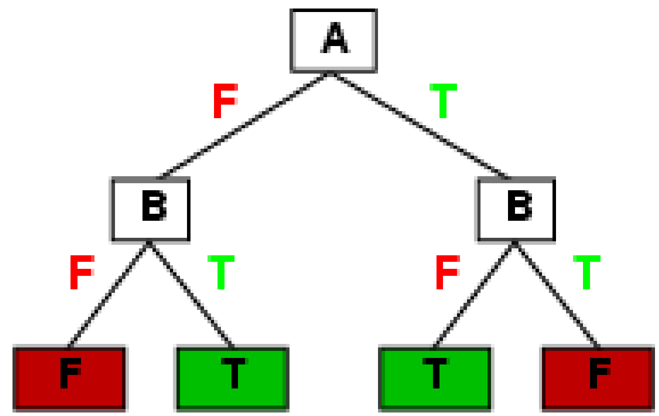
- Developed simultaneously by statistics and AI
- E.g., here is the "true" tree for deciding whether to wait:



Expressiveness

- Decision trees can express any function of the input attributes.
- E.g., for Boolean functions, truth table row → path to leaf:

A	B	A xor B
F	F	F
F	T	T
T	F	T
T	T	F



- Trivially, there is a consistent decision tree for any training set with one path to leaf for each example (unless *f* nondeterministic in *x*) but it probably won't generalize to new examples

- Prefer to find more **compact** decision trees

Hypothesis spaces

How many distinct decision trees with *n* Boolean attributes?

= number of Boolean functions

= number of distinct truth tables with 2^n rows = 2^{2^n}

- E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees

Hypothesis spaces

<u>How many distinct decision trees with *n* Boolean attributes?</u>
= number of Boolean functions
= number of distinct truth tables with 2^n rows = 2^{2^n}

- E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees

<u>How many purely conjunctive hypotheses (e.g., *Hungry* $\wedge$ $\neg$*Rain*)?</u>

- Each attribute can be in (positive), in (negative), or out
  $\Rightarrow$ 3^n distinct conjunctive hypotheses
- More expressive hypothesis space
  - increases chance that target function can be expressed
  - increases number of hypotheses consistent with training set
    $\Rightarrow$ may get worse predictions

The best hypothesis

- Find **best** function that models given data.

- How to define the best function?
  - Fidelity to the data – error on existing data: E(h,D)
  - Simplicity – how complicated is the solution: C(h)
    - One measure: how many possible hypotheses for the class?

Decision tree learning

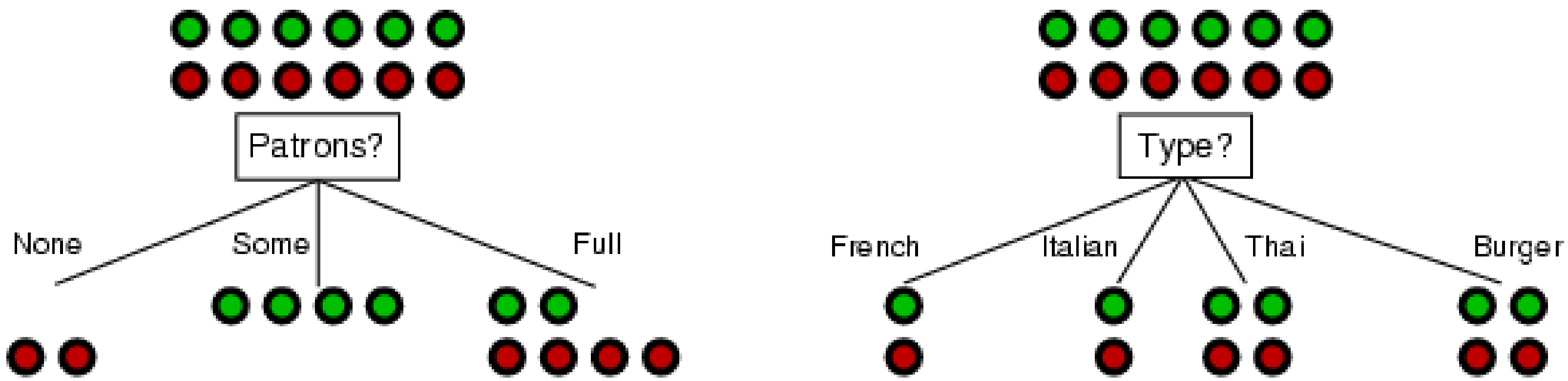
- Aim: find a small tree consistent with the training examples
- Idea: (recursively) choose "most significant" attribute as root of (sub)tree

function DTL(*examples*, *attributes*, *default*) **returns** a decision tree

    if *examples* is empty **then return** *default*
    else if all *examples* have the same classification **then return** the classification
    else if *attributes* is empty **then return** MODE(*examples*)
    else
        best ← CHOOSE-ATTRIBUTE(*attributes*, *examples*)
        tree ← a new decision tree with root test *best*
        for each value v_i of *best* **do**
            $examples_i$ ← {elements of *examples* with $best = v_i$}
            subtree ← DTL($examples_i$, *attributes* − *best*, MODE(*examples*))
            add a branch to *tree* with label v_i and subtree *subtree*
        return *tree*

Choosing an attribute

- Idea: a good attribute splits the training set into subsets that are (ideally) "all positive" or "all negative"



- *Patrons?* is a better choice

Information Content

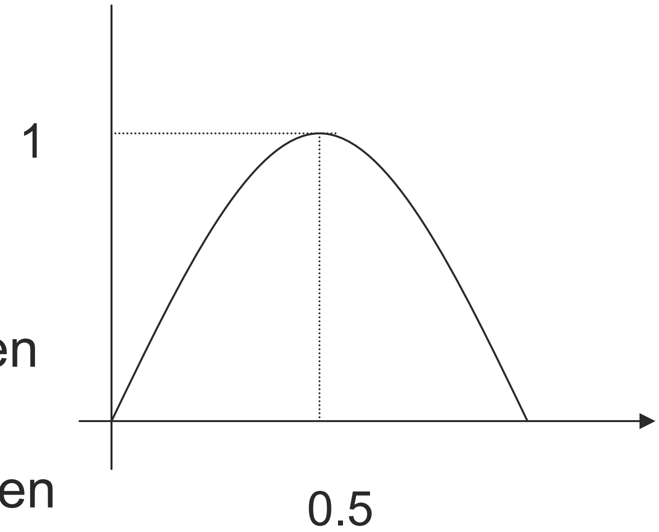
- **Entropy** measures purity of sets of examples
  - Normally denoted H(x)
- Or as **information content**: the less you need to know (to determine class of new case), the more information you have

T (total) = P + N

- With two classes (P,N):
  - $IC(S) = - (p/t) \log_2 (p/t) - (n/t) \log_2 (n/t)$
  - E.g., p=9, n=5;
    $IC([9,5]) = - (9/14) \log_2 (9/14) - (5/14) \log_2 (5/14) = 0.940$
  - Also, IC([14,0])=0; IC([7,7])=1

Entropy curve

- For p/t between 0 & 1, the 2-class entropy is
  - 0 when p/(p+n) is 0
  - 1 when p/(p+n) is 0.5
  - 0 when p/(p+n) is 1
  - monotonically increasing between 0 and 0.5
  - monotonically decreasing between 0.5 and 1
  - When the data is pure, only need to send 1 bit



Using information theory

- To implement **`Choose-Attribute`** in the DTL algorithm
- Entropy:

  $I(P(v_1), \ldots, P(v_n)) = \Sigma_{i=1} -P(v_i) \log_2 P(v_i)$
- For a training set containing *p* positive examples and *n* negative examples:

$$I\left(\frac{p}{p+n}, \frac{n}{p+n}\right) = -\frac{p}{p+n}\log_2\frac{p}{p+n} - \frac{n}{p+n}\log_2\frac{n}{p+n}$$

Information gain

- A chosen attribute *A* divides the training set *E* into subsets $E_1, \ldots, E_v$ according to their values for *A*, where *A* has v distinct values.

$$remainder(A) = \sum_{i=1}^{v} \frac{p_i + n_i}{p + n} I\left(\frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i}\right)$$

- Information Gain (IG) or reduction in entropy from the attribute test:

$$IG(A) = I\left(\frac{p}{p+n}, \frac{n}{p+n}\right) - remainder(A)$$

- Choose the attribute with the largest IG

Information gain

For the training set, $p = n = 6$, *I(6/12, 6/12) = 1* bit

Consider the attributes *Patrons* and *Type* (and others too):

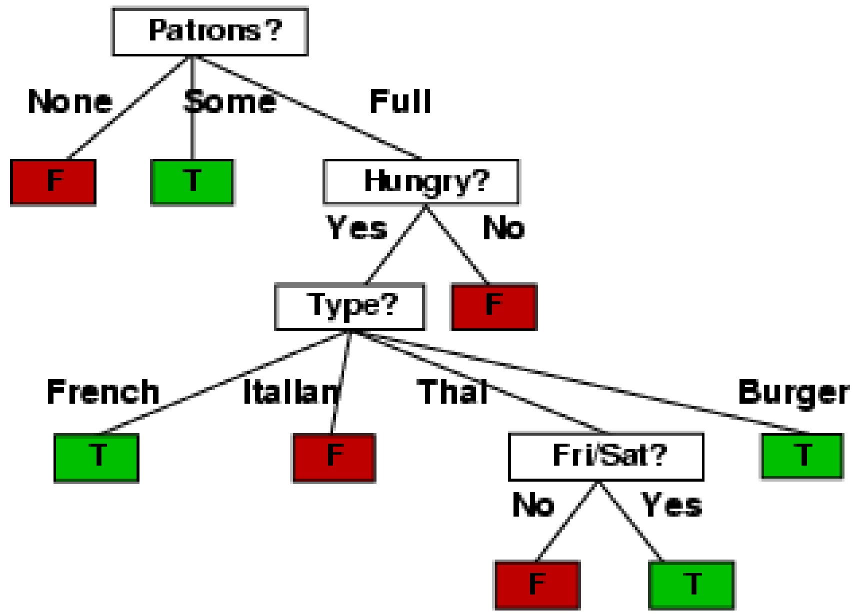
$$IG(Patrons) = 1 - [\frac{2}{12} I(0,1) + \frac{4}{12} I(1,0) + \frac{6}{12} I(\frac{2}{6}, \frac{4}{6})] = .0541 \text{ bits}$$

$$IG(Type) = 1 - [\frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4})] = 0 \text{ bits}$$

Patrons has the highest IG of all attributes and so is chosen by the DTL algorithm as the root

Example contd.

- Decision tree learned from the 12 examples:



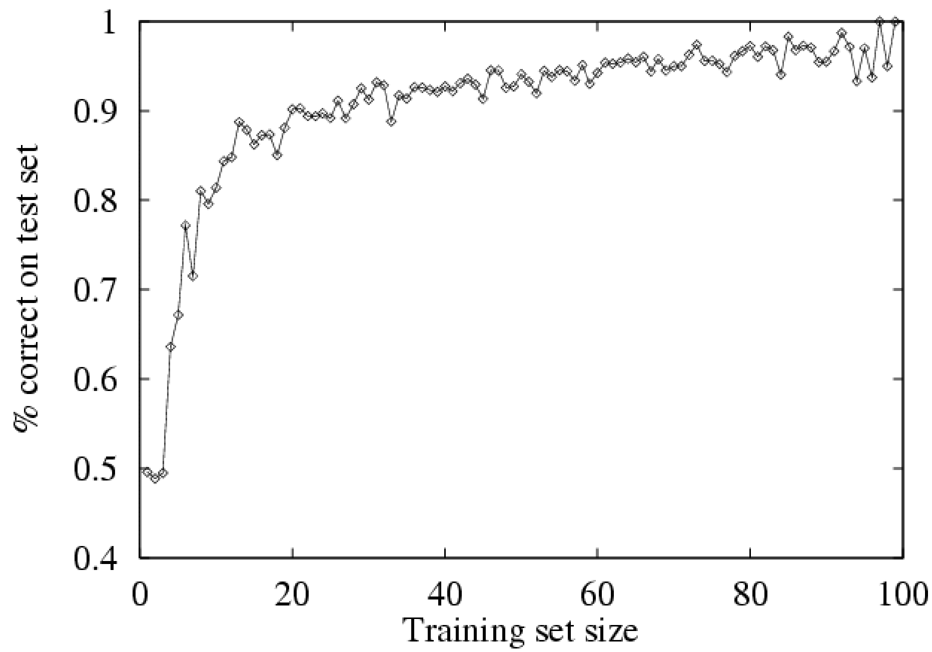
- Substantially simpler than “true” tree---a more complex hypothesis isn’t justified by small amount of data

Performance measurement

How do we know that $h \approx f$?

- Try h on a new **test set** of examples

Learning curve = % correct on test set as a function of training set size



Training and testing sets

- Where does the test set come from?
  1. Collect a large set of examples
  2. Divide into **training** and **testing data**
  3. Train on training data, assess on testing
  4. Repeat 1-3 for different splits of the set.

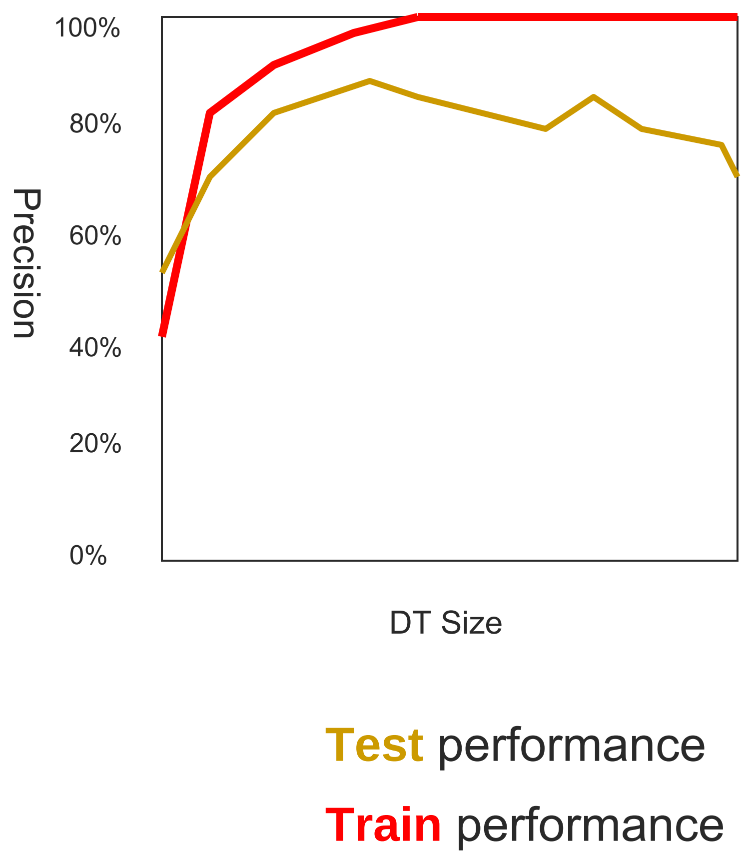
- Same distribution

  "Learning … enable[s] the system to do the task or tasks drawn from the **same population**" – Herb Simon

  - To think about: Why?

Overfitting

- Better training performance = test performance?
- Nope. Why?
  1. Hypothesis too specific
  2. Models noise
- Pruning
  - Keep complexity of hypothesis low
  - Stop splitting when:
    1. IC below a threshold
    2. Too few data points in node



Summary

- Learning needed for unknown environments, lazy designers
- Learning agent = performance element + learning element
- For supervised learning, the aim is to find a simple hypothesis approximately consistent with training examples
- Decision tree learning using information gain
- Learning performance = prediction accuracy measured on test set