CS3243 Foundations of Artificial Intelligence (2005/2006 Semester 2)
Tutorial 7

1. (a) (Question 8.2 from the textbook) Consider a knowledge base containing just two sentences: $P(a)$ and $P(b)$. Does this knowledge base entail $\forall x\ P(x)$? Explain your answer in terms of models.
   (b) (Question 8.3 from the textbook) Is the sentence $\exists x, y\ \ x = y$ valid? Explain.

2. Represent the following English sentences in first-order logic:
   (a) Anyone who meets the wumpus is killed by it.
   (b) Anything that glitters is gold.
   (c) Not every square contains a pit.

3. (Question 9.4 from the textbook) For each pair of atomic sentences, give the most general unifier if it exists:
   (a) P(A, B, B), P(x, y, z)
   (b) Q(y, G(A, B)), Q(G(x, x), y)
   (c) Older(Father(y), y), Older(Father(x), John)
   (d) Knows(Father(y), y), Knows(x, x)

4. An atheist asked two knowledge engineers to write a rule to say that "Nothing is divine!" The first engineer wrote $\neg\ \exists$x Divine(x) and transformed it into the following clause:

   $$\neg\ \text{Divine(G1)}$$

   where G1 is a Skolem constant. The second engineer wrote $\forall$x $\neg$ Divine(x) and transformed it into the following clause:

   $$\neg\ \text{Divine(x)}$$

   Why did they produce two different versions? Which version is correct?

5. Two English sentences "Anyone who takes an AI course is smart" and "Any course that teaches an AI topic is an AI course" have been represented in first-order logic:

   $$\forall x\ (\exists y\ \text{AI\_course}(y) \wedge \text{Takes}(x, y)) \Rightarrow \text{Smart}(x)$$

   $$\forall x\ (\exists y\ \text{AI\_topic}(y) \wedge \text{Teaches}(x, y)) \Rightarrow \text{AI\_course}(x)$$

   It is also known that John takes CS3243 and CS3243 teaches Inference which is an AI topic. Represent these facts as first-order logic sentences. Now convert all first-order logic sentences into conjunctive normal form and use resolution to prove that "John is smart".

6. (Slightly modified from Question 9.19 of the textbook) Here are two sentences in the language of first-order logic:
   (A): $\forall x\ \exists y\ (x \geq y)$
   (B): $\exists y\ \forall x\ (x \geq y)$
   (a) Assume that the variables range over all the natural numbers 0, 1, 2, … and that the ">" predicate means "is greater than or equal to." Under this interpretation, translate (A) and (B) into English.
   (b) Is (A) true under this interpretation? Is (B) true under this interpretation?
   (c) Does (A) logically entail (B)? Does (B) logically entail (A)? Justify your answers.