National University of Singapore
School of Computing
CS3243: Foundations of Artificial Intelligence
Tutorial 4

Readings: AIMA Chapter 5

1. Consider the `AC-3(csp)` algorithm (reproduced below), can the last line "add (X_k, X_i) to queue" be replaced with "if $X_k \neq Xj$ then add (X_k, X_i) to queue"? Justify your answer.

```
function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X1, X2, ..., Xn}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (Xi, Xj) ← REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(Xi, Xj) then
         for each Xk in NEIGHBORS[Xi] do
            add (Xk, Xi) to queue
---------------------------------------------------------------------
function REMOVE-INCONSISTENT-VALUES( Xi, Xj) returns true iff succeeds
   removed ← false
   for each x in DOMAIN[Xi] do
      if no value y in DOMAIN[Xj] allows (x,y) to satisfy the constraint Xi ↔ Xj
         then delete x from DOMAIN[Xi];  removed ← true
   return removed
```

2. Consider the following constraint satisfaction problem:

   Variables:
   $$A, B, C$$
   Domains:
   $$D_A = D_B = D_C = \{0, 1, 2, 3, 4\}$$
   Constraints:
   $$\begin{aligned} A &= B + 1 \\ B &= 2C \end{aligned}$$

   Construct a constraint graph for this problem. Show a trace of the `AC-3` algorithm on this problem. Assume that initially, the arcs in queue are in the order $\{(A, B), (B, A), (B, C), (C, B)\}$.

3. Consider the 4-queens problem on a 4×4 chess board. Suppose the leftmost column is column 1, and the topmost row is row 1. Let Q_i denote the row number of the queen in column i, $i = 1, 2, 3, 4$. Assume that variables are assigned in the order Q_1, Q_2, Q_3, Q_4, and the domain values of Q_i are tried in the order $1, 2, 3, 4$. Show a trace of the backtracking algorithm with forward checking to solve the 4-queens problem.

```
    T W O
+   T W O
---------
  F O U R
```

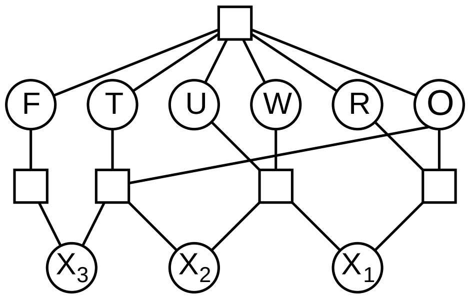

Figure 1: Cryptarithmetic puzzle.

4. Show a trace of the backtracking algorithm with forward checking to solve the cryptarithmetic problem shown in Figure 1. Use the most constrained variable heuristic, and assume that the domain values (digits) are tried in ascending order (i.e., 0, 1, 2, $\cdots$).