National University of Singapore
School of Computing
CS3243: Foundations of Artificial Intelligence
Tutorial 7

Readings: AIMA Chapter 9

1. (Question 9.4 from AIMA) For the following pairs of atomic sentences, give the most general unifier if one exists:

   (a) $P(A, B, B)$, $P(x, y, z)$
   (b) $Q(y, G(A, B))$, $Q(G(x, x), y)$
   (c) $Older(Father(y), y)$, $Older(Father(x), John)$
   (d) $Knows(Father(y), y)$, $Knows(x, x)$

2. What is the problem in each the following first order logic statements? Suggest how these statements can be corrected.

   (a) $\forall x\ Boy(x) \wedge Tall(x)$
   (Intended meaning: all boys are tall.)
   (b) $\exists x\ Boy(x) \Rightarrow Tall(x)$
   (Intended meaning: some boy is tall.)

3. An atheist asked two knowledge engineers to write a rule to say that Nothing is divine! The first engineer wrote $\neg\exists x\ Divine(x)$ and transformed it into the following clause:

   $$\neg Divine(G1)$$

   where $G1$ is a Skolem constant. The second engineer wrote $\forall x\ \neg Divine(x)$ and transformed it into the following clause:

   $$\neg Divine(x)$$

   Why did they produce two different versions? Which version is correct?

4. Two English sentences "Anyone who takes an AI course is smart" and "Any course that teaches an AI topic is an AI course" have been represented in first-order logic:

   $$\forall x\ (\exists y\ AI\_course(y) \wedge Takes(x, y)) \Rightarrow Smart(x)$$

   $$\forall x\ (\exists y\ AI\_topic(y) \wedge Teaches(x, y)) \Rightarrow AI\_course(x)$$

   It is also known that John takes CS3243 and CS3243 teaches Inference which is an AI topic. Represent these facts as first-order logic sentences. Now convert all first-order logic sentences into conjunctive normal form and use resolution to prove that "John is smart."

5. (Slightly modified from Question 9.19 of AIMA) Here are two sentences in the language of first-order logic:
   $(A) : \forall x\ \exists y\ (x \geq y)$
   $(B) : \exists y\ \forall x\ (x \geq y)$

(a) Assume that the variables range over all the natural numbers $0, 1, 2, \cdots$ and that the ">" predicate means "is greater than or equal to." Under this interpretation, translate (A) and (B) into English.

(b) Is (A) true under this interpretation? Is (B) true under this interpretation?

(c) Does (A) logically entail (B)? Does (B) logically entail (A)? Justify your answers.

6. (old final exam problem) PSA would like to implement its tax system on ships and cargo for its Brani port as part of a first order logic system. You have been hired as a knowledge engineer to convert the following predicates into FOL representation. You may use any predicate that you create in previous parts for subsequent parts. You may also define new constants and predicates.

   Note that variables are in lowercase, whereas constant, predicate and function symbols start with uppercase. Given the functions:

   $Arrival\_Time(ship)$
   $Departure\_Time(ship)$

   And the predicates:

   $Unload\_From\_Ship(cargo, ship, arrival\_time)$
   $Load\_Onto\_Ship(cargo, ship, departure\_time)$
   $Weapons(cargo)$

   (a) In the PSA system, can ship objects (n.b., not constant symbols) arrive at the Brani port multiple times? Justify your answer.

   (b) Write a FOL predicate of a simplified tax law for cargos entering and not departing Singapore. Note that ships unload and load at their arrival and departure times.

   (c) Aside from cargo, ships are also taxed. A ship is taxable upon entry to Singapore unless all the ship's cargo are weapons (slated for the armed forces).

   (d) State your answer from Part (c) in Conjunctive Normal Form.

   (e) Using the following observations, use resolution by refutation to answer the query $Taxable(Storm\_King)$:

$$Unload\_From\_Ship(Torpedos, Storm\_King, Arrival\_Time(Storm\_King))$$

$$Weapons(Torpedo)$$

$$Unload\_From\_Ship(Laser\_Parts, Storm\_King, Arrival\_Time(Storm\_King))$$

$$\neg Weapons(Laser\_Parts)$$