National University of Singapore
School of Computing
CS3243: Foundations of Artificial Intelligence
Solutions for Tutorial 5

Readings: AIMA Chapter 7

1. Determine using a truth table whether the following sentence is valid, satisfiable, or unsatisfiable:

   (a) $(P \wedge Q) \vee \neg Q$

   Construct the following truth table:

P	Q	$P \wedge Q$	$\neg Q$	$(P \wedge Q) \vee \neg Q$
F	F	F	T	T
F	T	F	F	F
T	F	F	T	T
T	T	T	F	T

   Since the sentence $(P \wedge Q) \vee \neg Q$ is true in at least one model but not all models, the sentence is satisfiable.

   (b) $((P \wedge Q) \Rightarrow R) \Leftrightarrow ((P \Rightarrow R) \vee (Q \Rightarrow R))$

   Construct the following truth table:

P	Q	R	$P \wedge Q$	$(P \wedge Q) \Rightarrow R \equiv \alpha$	$P \Rightarrow R$	$Q \Rightarrow R$	$(P \Rightarrow R) \vee (Q \Rightarrow R) \equiv \beta$	$\alpha \Leftrightarrow \beta$
F	F	F	F	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	T	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
T	F	T	F	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	T	T	T	T	T	T	T	T

   Since the sentence $((P \wedge Q) \Rightarrow R) \Leftrightarrow ((P \Rightarrow R) \vee (Q \Rightarrow R))$ is true in all models, the sentence is valid.

2. Assume that a knowledge base KB contains the following rules:

$$\begin{aligned} poor &\Rightarrow \neg worried \\ rich &\Rightarrow scared \\ \neg rich &\Rightarrow poor \end{aligned}$$

   (a) Show that $KB \models (worried \Rightarrow scared)$, using the model checking approach.

We define:

$$\begin{aligned} A &\equiv P \Rightarrow \neg W \\ &\equiv \neg P \vee \neg W \\ B &\equiv R \Rightarrow S \\ &\equiv \neg R \vee S \\ C &\equiv \neg R \Rightarrow P \\ &\equiv R \vee P \end{aligned}$$

where $W = worried, S = scared, P = poor, R = rich$
Construct the following truth table:

W	S	P	R	A $(\neg P \vee \neg W)$	B $(\neg R \vee S)$	C $(R \vee P)$	KB	$W \Rightarrow S$
F	F	F	F	T	T	F	F	T
F	F	F	T	T	F	T	F	T
F	F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	F	T
F	T	F	F	T	T	F	F	T
F	T	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	F	F
T	F	F	T	T	F	T	F	F
T	F	T	F	F	T	T	F	F
T	F	T	T	F	F	T	F	F
T	T	F	F	T	T	F	F	T
T	T	F	T	T	T	T	T	T
T	T	T	F	F	T	T	F	T
T	T	T	T	T	T	T	F	T

From the above truth table, we find that in every model in which KB *is true,* $W \Rightarrow S$ *is also true. Thus* KB *entails* $W \Rightarrow S$.

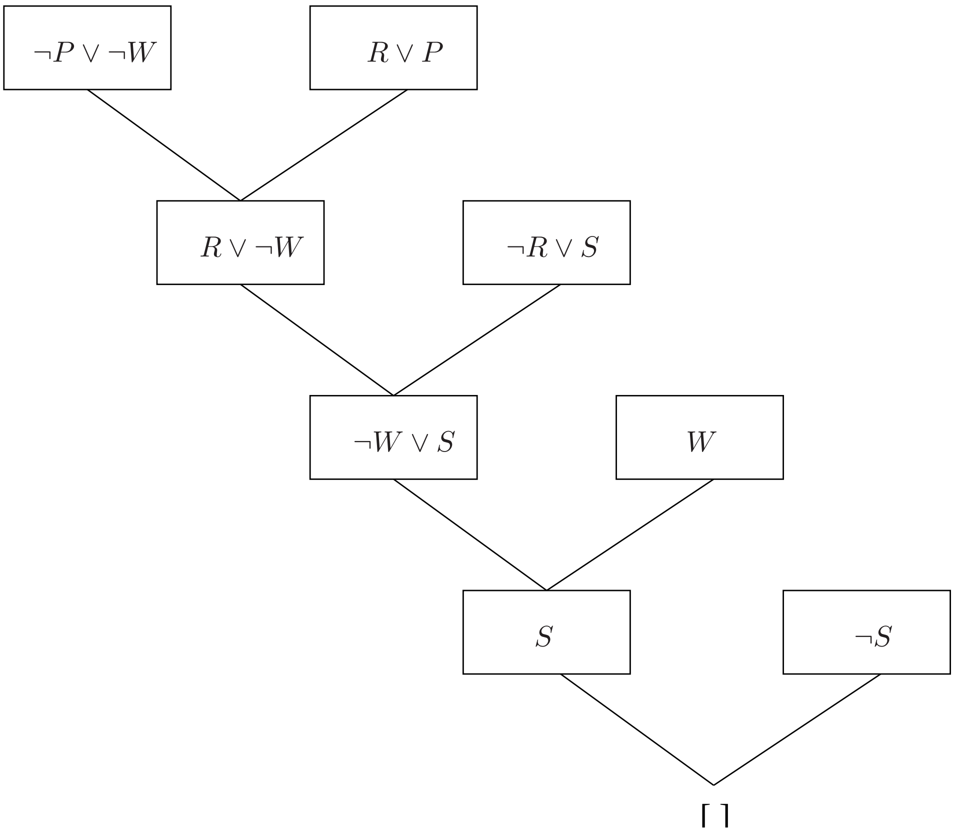
(b) Use resolution to prove $KB \models (worried \Rightarrow scared)$.
Using resolution, we add the negation of the conclusion to the KB, which is $\neg(W \Rightarrow S)$. However,

$$\begin{aligned} \neg(W \Rightarrow S) &\equiv \neg(\neg W \vee S) \\ &\equiv W \wedge \neg S \end{aligned}$$

So we add two terms W and $\neg S$ to the KB and run the resolution algorithm as follows:

Table 1: Table of logical equivalences

$(\alpha \land \beta)$	$\equiv$	$(\beta \land \alpha)$	commutativity of $\land$
$(\alpha \lor \beta)$	$\equiv$	$(\beta \lor \alpha)$	commutativity of $\lor$
$((\alpha \land \beta) \land \gamma)$	$\equiv$	$(\alpha \land (\beta \land \gamma))$	associativity of $\land$
$((\alpha \lor \beta) \lor \gamma)$	$\equiv$	$(\alpha \lor (\beta \lor \gamma))$	associativity of $\lor$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \lor \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \land \beta)$	$\equiv$	$(\neg\alpha \lor \neg\beta)$	De Morgan
$\neg(\alpha \lor \beta)$	$\equiv$	$(\neg\alpha \land \neg\beta)$	De Morgan
$(\alpha \land (\beta \lor \gamma))$	$\equiv$	$((\alpha \land \beta) \lor (\alpha \land \gamma))$	distributivity of $\land$ over $\lor$
$(\alpha \lor (\beta \land \gamma))$	$\equiv$	$((\alpha \lor \beta) \land (\alpha \lor \gamma))$	distributivity of $\lor$ over $\land$



3. Someone says: "On either Saturday or Sunday, if I am free, I will go to the concert". Using propositional logic, the statement is represented as:

$$(saturday \lor sunday) \Rightarrow (free \Rightarrow concert)$$

Convert the above sentence into conjunctive normal form, and then into Horn form, by using the logical equivalences shown in Table **??**.

Applying the logical equivalences:

$$(saturday \lor sunday) \Rightarrow (free \Rightarrow concert)$$
$$\neg(saturday \lor sunday) \lor (\neg free \lor concert)$$
$$(\neg saturday \land \neg sunday) \lor (\neg free \lor concert)$$
$$(\neg saturday \lor \neg free \lor concert) \land (\neg sunday \lor \neg free \lor concert)$$

The last sentence is in conjunctive normal form (CNF). Since each clause has at most one positive literal, the last sentence is also in Horn form, i.e.,

$$(\neg saturday \lor \neg free \lor concert) \land (\neg sunday \lor \neg free \lor concert)$$
$$(saturday \land free \Rightarrow concert) \land (sunday \land free \Rightarrow concert)$$

4. (Question 7.9 from AIMA) (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

    If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

    Let
    Y*: the unicorn is mythical*
    O*: the unicorn is mortal*
    M*: the unicorn is a mammal*
    H*: the unicorn is horned*
    G*: the unicorn is magical*

    KB:

    - $Y \Rightarrow \neg O$ *(If the unicorn is mythical, then it is immortal)*
    - $\neg Y \Rightarrow (O \land M)$ *(if it is not mythical, then it is a mortal mammal)*
    - $(\neg O \lor M) \Rightarrow H$ *(If the unicorn is either immortal or a mammal, then it is horned])*
    - $H \Rightarrow G$ *(The unicorn is magical if it is horned)*

    Converting these statements to to CNF, we obtain the following clauses:
    (1) $\neg Y \lor \neg O$
    (2) $Y \lor O$
    (3) $Y \lor M$
    (4) $O \lor H$
    (5) $\neg M \lor H$
    (6) $\neg H \lor G$

    (a) To prove that the unicorn is mythical (Y), we add the negation ($\neg Y$) and attempt to derive a contradiction using resolution:
    (7) $\neg Y$
    (8) O *[(7) & (2)]*
    (9) M *[(7) & (3)]*
    (10) H *[(9) & (5)]*
    (11) G *[(10) & (6)]*
    As it turns out, we cannot derive a contradiction. In particular, in the model $\{Y \leftarrow false, O \leftarrow true, M \leftarrow true, H \leftarrow true, G \leftarrow true\}$*,* KB *is true and* Y *is false. Hence,* Y *is not entailed by the* KB*.*

(b) *To prove that the unicorn is magical (G), we add the negation ($\neg G$) and derive a contradiction using resolution:*

(7)	$\neg G$	
(8)	$\neg H$	[(7) & (6)]
(9)	$\neg M$	[(8) & (5)]
(10)	Y	[(9) & (3)]
(11)	$\neg O$	[(10) & (1)]
(12)	H	[(11) & (4)]
(13)	[ ]	[(12) & (8)]

(c) *To prove that the unicorn is horned (H), we add the negation ($\neg H$) and derive a contradiction using resolution:*

(7)	$\neg H$	
(8)	$\neg M$	[(7) & (5)]
(9)	Y	[(8) & (3)]
(10)	$\neg O$	[(9) & (1)]
(11)	H	[(10) & (4)]
(12)	[ ]	[(12) & (8)]

5. (Question 7.6 from AIMA) We have defined four different binary logical connectives (namely $\wedge, \vee, \Rightarrow, \Leftrightarrow$).

   (a) Are there any others that might be useful?

   (b) How many binary connectives can there be?

   (c) Why are some of them not very useful?

	$P=F,Q=F$	$P=F,Q=T$	$P=T,Q=F$	$P=T,Q=T$
P	F	F	T	T
Q	F	T	F	T
False	F	F	F	F
$P \wedge Q$	F	F	F	T
$\neg(P \Rightarrow Q)$	F	F	T	F
P	F	F	T	T
$\neg(Q \Rightarrow P)$	F	T	F	F
Q	F	T	F	T
P XOR Q	F	T	T	F
$P \vee Q$	F	T	T	T
P NOR Q	T	F	F	F
$P \Leftrightarrow Q$	T	F	F	T
$\neg Q$	F	F	T	F
$Q \Rightarrow P$	T	F	T	T
$\neg P$	T	T	F	F
$P \Rightarrow Q$	T	T	F	T
P NAND Q	T	T	T	F
True	T	T	T	T

From the above table, we see that there are 16 distinct binary connectives. Some of them are useful (e.g., XOR, NAND, NOR). Others are not useful (e.g., P, Q, True, False), since they ignore one or both inputs.