National University of Singapore
School of Computing

The Tourist Problem:
And Fun with Graph Modelling

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The Tourist Problem

❑ Organization

- ❖ The Tourist Problem
- ❖ Analysis and Simplifications
- ❖ Problem Modelling (with *Graphs*)
- ❖ Solving the *Graph* Model
- ❖ Mapping back the Solution
- ❖ Moral of the Story

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The Tourist Problem…

Given: **A list of tourist, each with his/her list of places to visit.**

To do: **Schedule bus rides for them so that each tourist visits all the places in his/her list.**

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG
Frances	BG, SZG, JB
Gary	CG, OR
Harry	JG, CG

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The Tourist Problem (Entities)

- Good to know the entities we are dealing with…
  - The Tourists:
    $T = \{ A, B, C, D, E, F, G, H \}$
  - The Attractions (Places):
    $P = \{ BG, CG, JB, JG, OR, SI, VC, SZG \}$

… Places of Attraction …

Place	Common Name	Place	Common Name
BG	Botanical Gardens	CG	Chinese Gardens
JB	Jurong Birdpark	JG	Japanese Gardens
OR	Orchard Road	SI	Sentosa Island
SZG	Spore Zoological Gardens	VC	VivoCity

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The Tourist Problem (Analysis…)

Some Simplifications: Consider

- ❖ Aaron { SZG, BG, JB }
  ~~Frances { SZG, BG, JB }~~

Also consider

- ❖ David { JG, CG, OR }
  ~~Gary { CG, OR }~~

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG
~~Frances~~	~~BG, SZG, JB~~
~~Gary~~	~~CG, OR~~
~~Harry~~	~~JG, CG~~

Simplification Rule:

If $P(T_1) \subseteq P(T_2)$, then tourist T_1 can just "follows" tourist T_2. Thus, we can omit T_1 from consideration.

Oh, can also omit Harry

- ❖ Betty { CG, JG, BG }
  ~~Harry { CG, JG }~~

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The (Reduced) Tourist Problem…

Given: A list of tourist, each with his/her list of places to visit.

To do: Schedule bus rides for them so that each tourist visits all the places in his/her list.

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

$T = \{ A, B, C, D, E \}$

$P = \{ BG, CG, JB, JG, OR, SI, VC, SZG \}$

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The Tourist Problem – v0

Given: A list of tourist, each with his/her list of places to visit.

To do: Schedule bus rides for them so that
each tourist visits all the places in his/her list.

Solution: (Singapore 1-Day Tour)

Put all the tourists on one bus.
Visit all eight places in 1 day.

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

What's Good: It works! One bus, one-day.

What's Bad: Too rushed. NO time to see anything!

Not interesting!

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The Tourist Problem – v0.5

Given: **A list of tourist, each with his/her list of places to visit.**

To do: **Schedule bus rides for them so that**
each tourist visits all the places in his/her list, *and*

C1: Each tourist visits *at most one place a day*.

Simple Solution:

Schedule *one trip to every place every day*.

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

What's Good: **It works! Finish in 3 days. (*minimum*!)**

What's Bad: **Wasteful! 24 bus trips.**

Also, not so interesting!

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The Tourist Problem – v0.8

Given: A list of tourist, each with his/her list of places to visit.

To do: Schedule bus rides for them so that
each tourist visits all the places in his/her list,

C1: Each tourist visits *at most one place a day*, *and*

C2: There is *at most one bus trip to each place*

Simple Solution:

Schedule *one trip per day*,
each to a *different* place.

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

What's Good: It works! 8 trips.

What's Bad: It takes 8 days!

But wait… Did you see something interesting?

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The Tourist Problem – v1.0

Given: **A list of tourist, each with his/her list of places to visit.**

To do: **Schedule bus rides for them so that**
each tourist visits all the places in his/her list,
C1: Each tourist visits *at most one place a day*,
C2: There is *at most one bus trip to each place*, *and*
C3: *minimize* the number of days to complete mission.

Observation:

On the same day,
cannot schedule SZG and BG
can schedule SZG and OR

How to model all these constraints?

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

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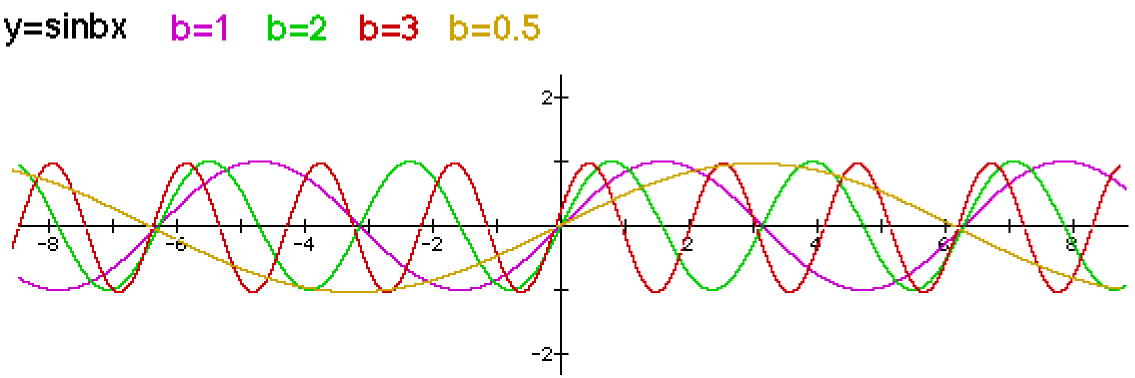
The Graph Model

- **What is a graph?**
  - **eg: $y = \sin(bx)$**



- **No. Not this type of graph.**

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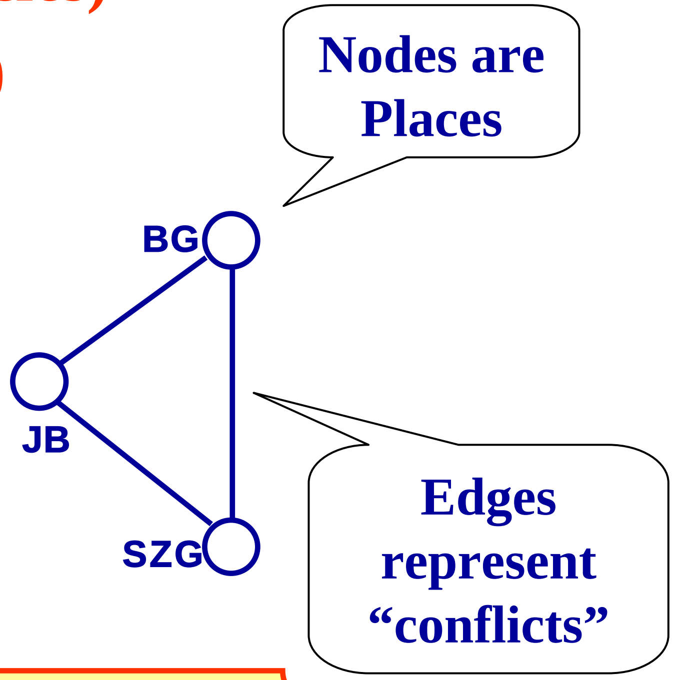
The Graph Model

- Graph $G = (V, E)$
  - V is a set of vertices, nodes (circles)
  - E is a set of edges (connections)

An Instance of Tourist Problem	
Tourist	**Places of Interest**
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG



In our graph, nodes are places, and edges in the graph means conflicts.

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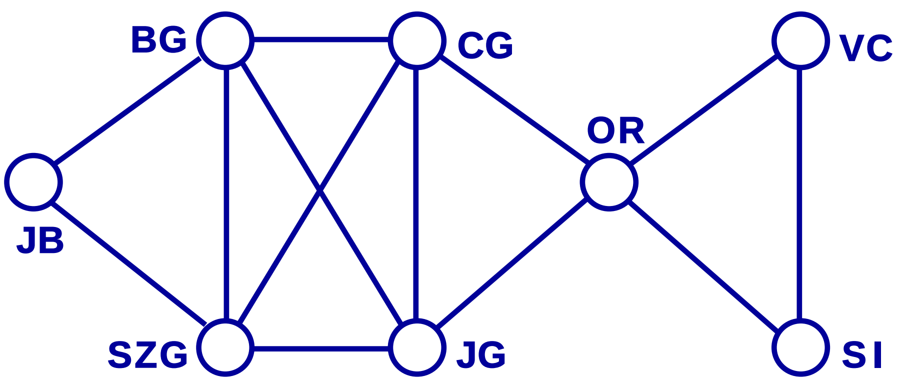
Graph Model for the Tourist Problem

An Instance of Tourist Problem	
Tourist	**Places of Interest**
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

***G* = (*V*, *E*)**



The graph G = (V, E) captures all the conflicts for our tourist problem instance.

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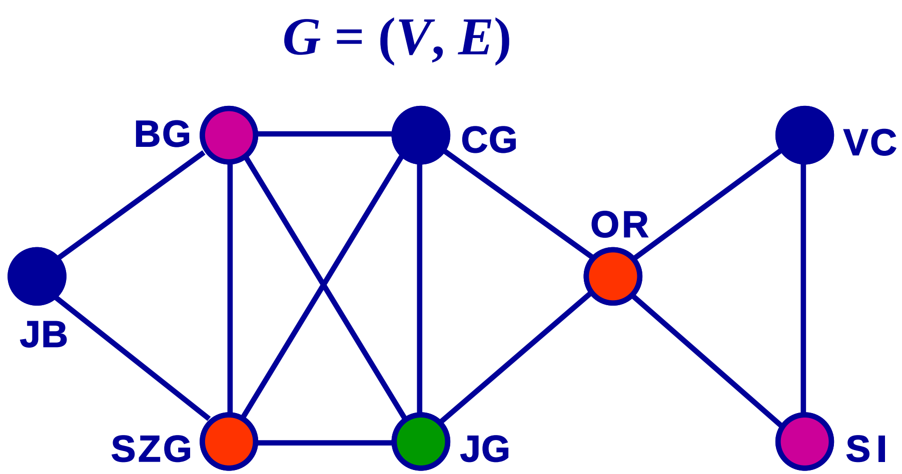
Graph Model for the Tourist Problem

- **What's good about the graph model?**
  - *very simple* !
  - easy to spot conflicts and *non-conflicts*



On Day 1, can schedule SZG, OR [Any more? Why?]
On Day 2, can schedule JB, CG, VC
On Day 3, can schedule BG, SI
On Day 4, can schedule JG

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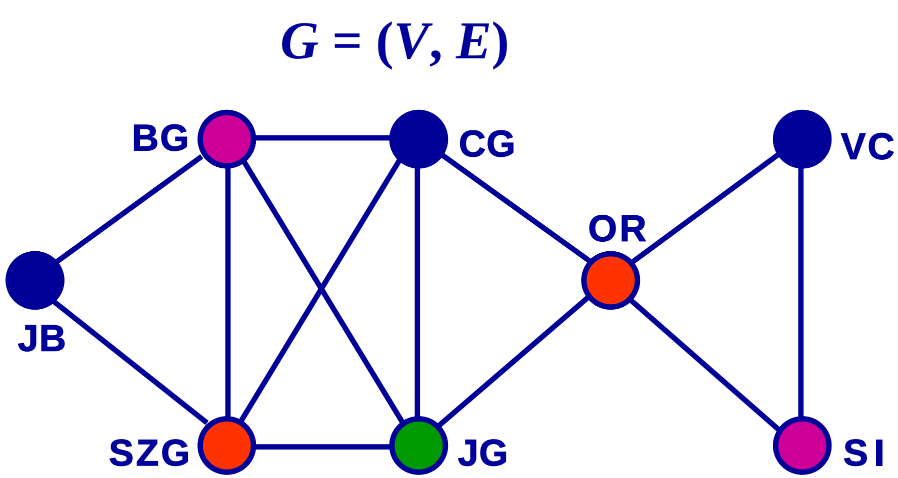
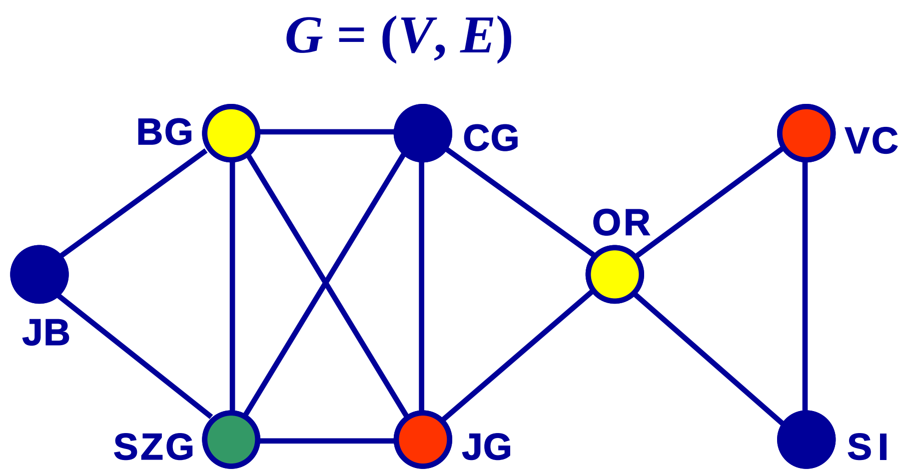
Graph Coloring Problem

- Given a graph $G = (V, E)$, colour the vertices in V so that any two vertices that are connected by an edge in E will have *different* colors.

  We want to *minimize* the number of colors.





Number of colors used to color the graph G	=	*Number of days needed to complete the schedule*

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Activity Period #1:

Graph Colouring Exercises

(10 minutes)

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Review of Activity

- **Is Graph Colouring fun?**
  - **Did you *really* used different colours?**
- **How many colours was did you use?**
  - **Q1 and Q2**
- **What about the cycles (Q3):**
  - **Q3(a) C_6 (a cycle of length 6)?**
  - **C_6 (a cycle of length 6)?**
- **What about the final graph?**

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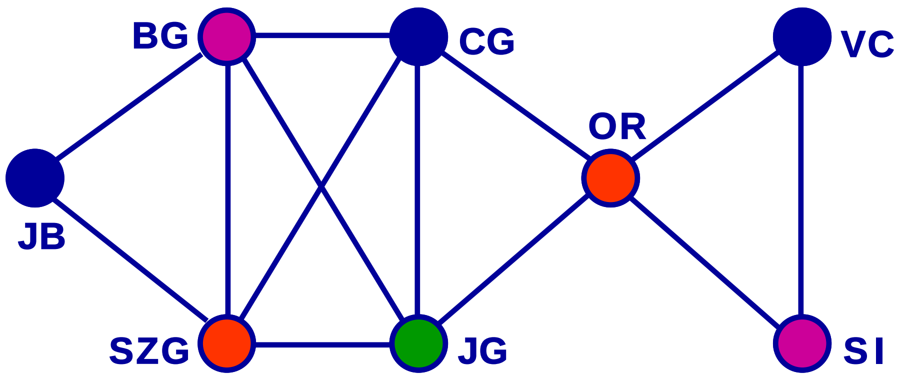
Get Solution to Tourist Problem - 1

- ❑ **Colored graph ⇒ "Bus Schedule"**

$G = (V, E)$



COLORED GRAPH

Color	Day	Place
(red-orange)	1	SZG, OR
(dark blue)	2	JB, CG, VC
(magenta)	3	BG, SI
(green)	4	JG

1. What about the list of tourists on each bus?

Can we get it from the graph model?

NO. ***Why*** **NOT.**

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The Tourist Problem…

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG
Frances	BG, SZG, JB
Gary	CG, OR
Harry	JG, CG

Alternative Representation

Tourist	BG	CG	JB	JG	OR	SI	SZG	VC
Aaron	X		X				X	
Betty	X	X		X				
Cathy					X	X		X
David		X		X	X			
Evans		X		X			X	
Frances	X		X				X	
Gary		X			X			
Harry		X		X				

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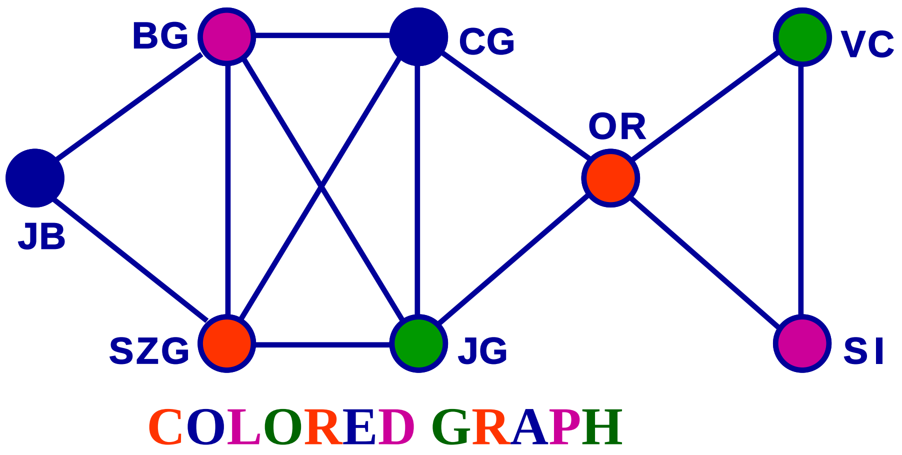
Get Solutions to Tourist Problem (2)

- ❑ **Colored graph ⇒ "Bus Schedule"**

$G = (V, E)$



Color	Day	Place
(red)	1	SZG, OR
(blue)	2	JB, CG, ~~VC~~
(magenta)	3	BG, SI
(green)	4	JG, VC

1. What about the list of tourists on each bus?
2. What if you only have 2 buses?
   - can color vertex VC green.

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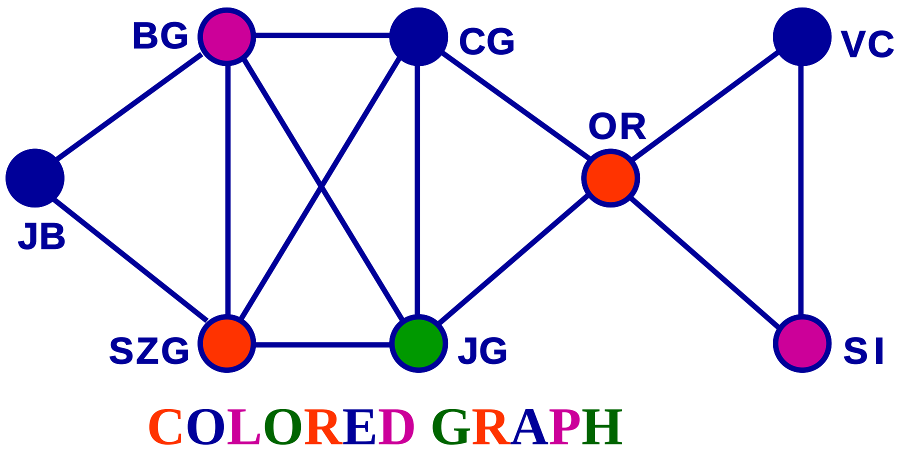
Get Solutions to Tourist Problem (3)

- ❑ **Colored graph ⇒ "Bus Schedule"**

$G = (V, E)$



Color	Day	Place
(orange)	1	SZG, OR
(dark blue)	2	JB, CG, VC
(magenta)	3	BG, SI
(green)	4	JG

1. What about the list of tourists on each bus?
2. What if you only have 2 buses?
3. Can we re-order the colors?
4. Can we use fewer colors (fewer days)?

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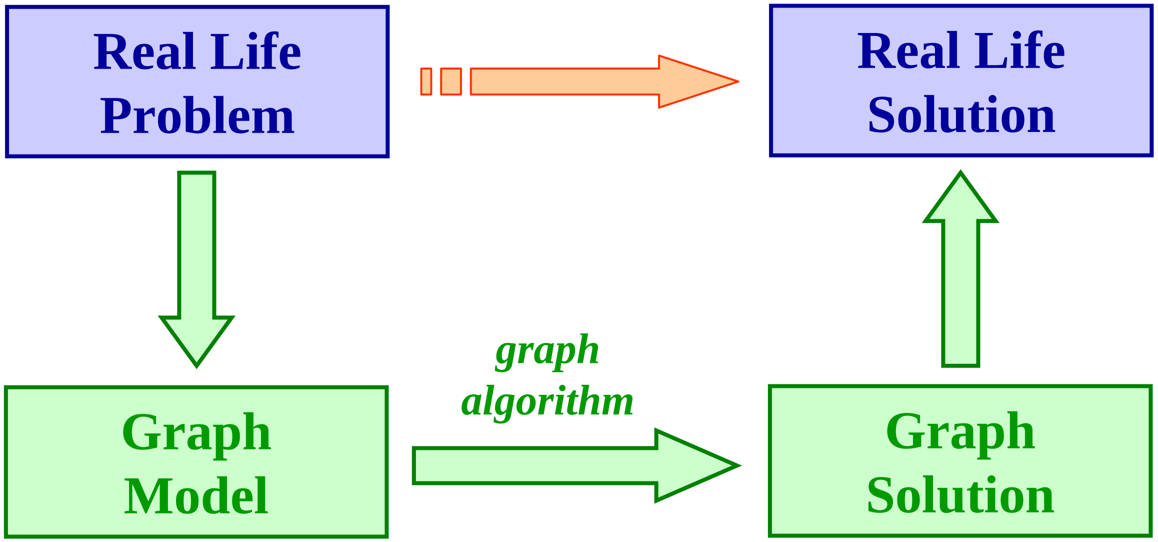
Graph Modelling…



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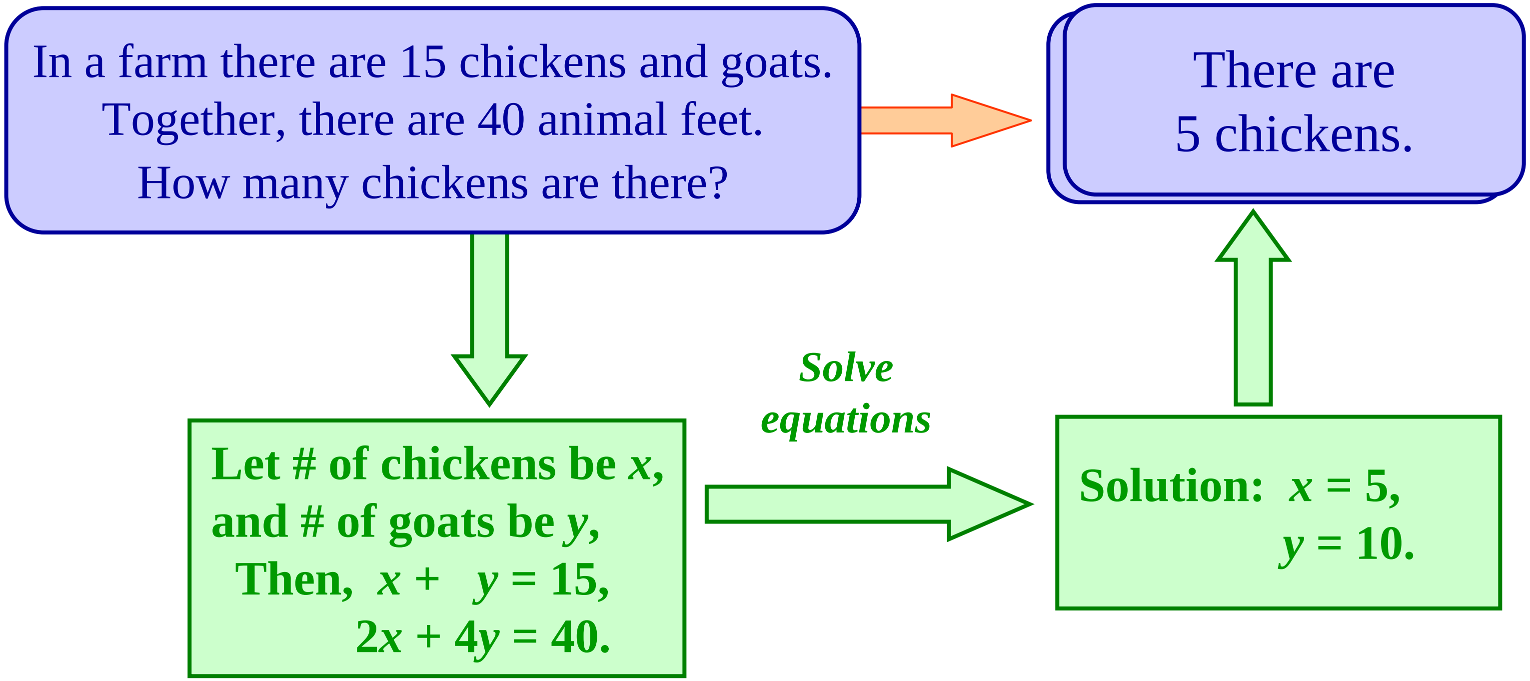
Modelling…

- **Nothing new. You do it *all* the time.**



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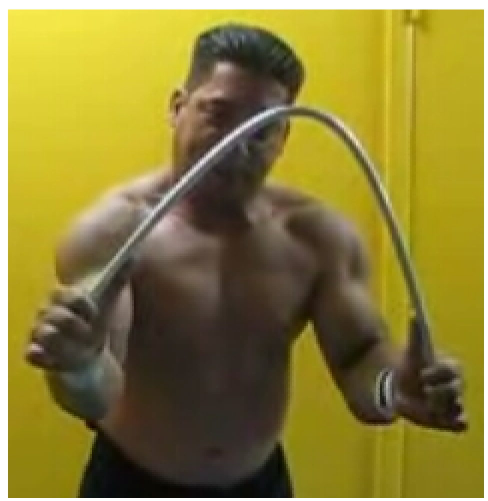
Modelling: Another example

- **Bend a steel bar**

(Direct method)



Man bending steel rod

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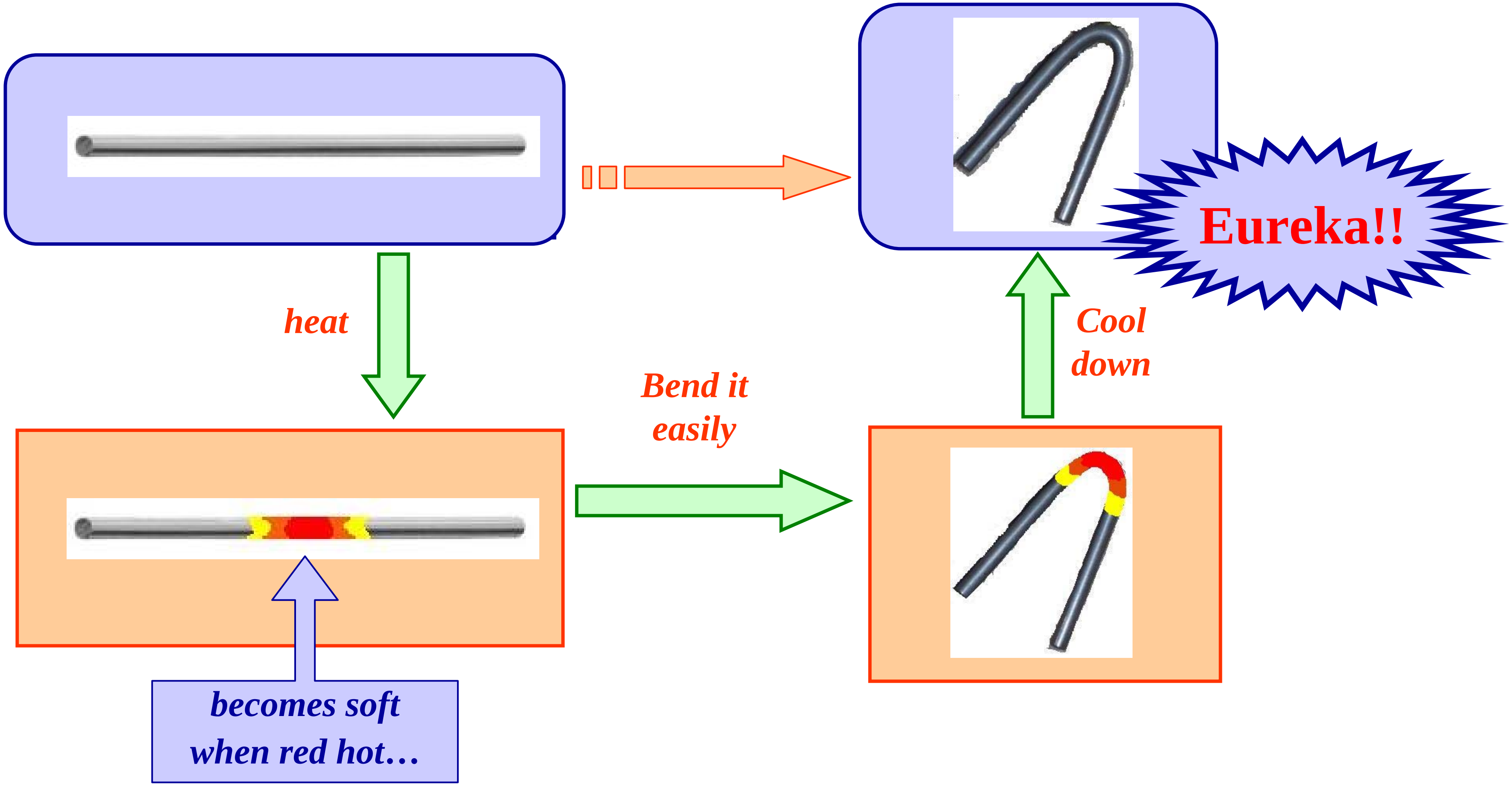
Modelling: Another example (2)

- Bend a steel bar (using *transformation*)



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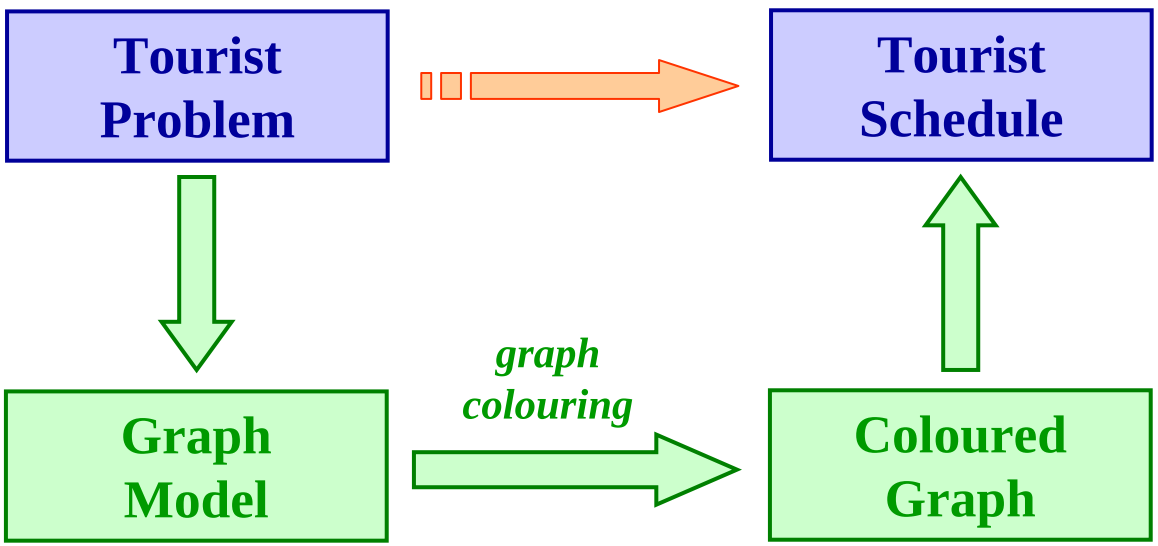
Tourist Problem & Graph Colouring



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Modelling in Tourist Problem

Recap: Our Graph modelling…

Graph Model	Tourist Problem			
Nodes	*places*			
Edges / Conflicts	*tourist want to visit both places*			
Colors	*bus trips to places*			
Others	*The tourists*			

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Moral of the Story

- **The Tourist Problem:**
  - **Some problems are EASY. (don't complicate them)**
  - **Get a *simple* solution first.**

    then *analyze* it, *improve* it, *refine* it.
  - **Solution depend on the questions asked**
  - **It is important to ASK QUESTIONS.**
  - **Theoretical modelling and analysis are beneficial**
- **Modelling**
  - **Abstract modeling simplifies problem and solution!**
  - **Abstract model is transferable.**
  - **Models don't answer everything.**

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Graph Coloring & Applications

- **Where *else* is Graph Coloring used?**
  - **The Tourist Problem [done]**
  - **Map Coloring**
  - **Fish in a Tank**
  - **Frequency assignment in wireless networks**
  - **Time Table Scheduling**
  - **And a whole lot more…**

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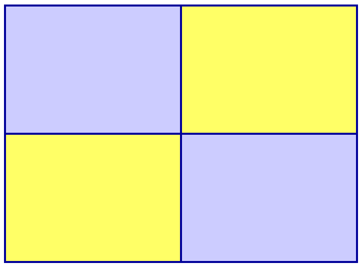
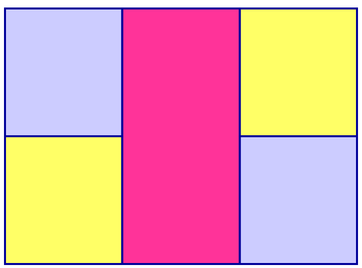
The Map Coloring Problem

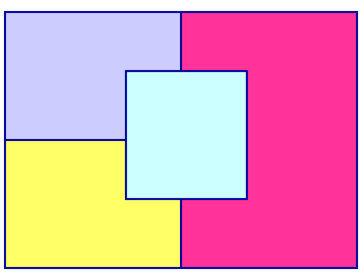
We want to color countries, oceans, lakes, and islands on a map so that no two adjacent areas have the same color.



Two colors



Three colors



Four colors

Brock University Map Library

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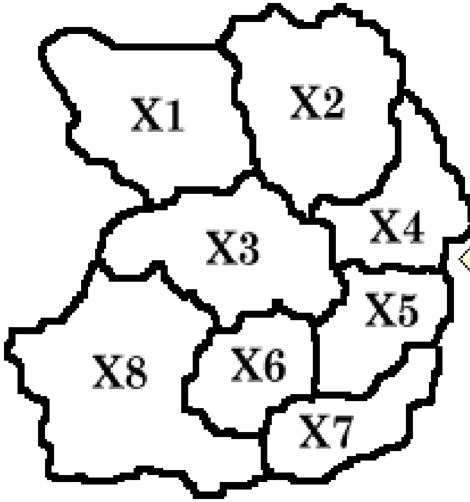
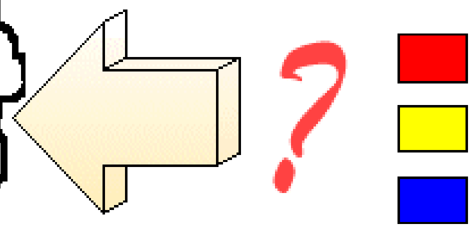
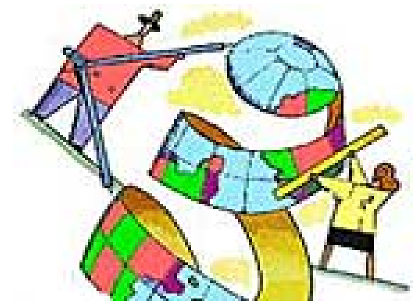
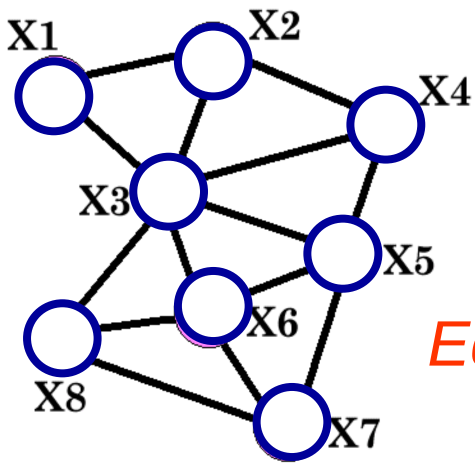
Map and Graph Coloring



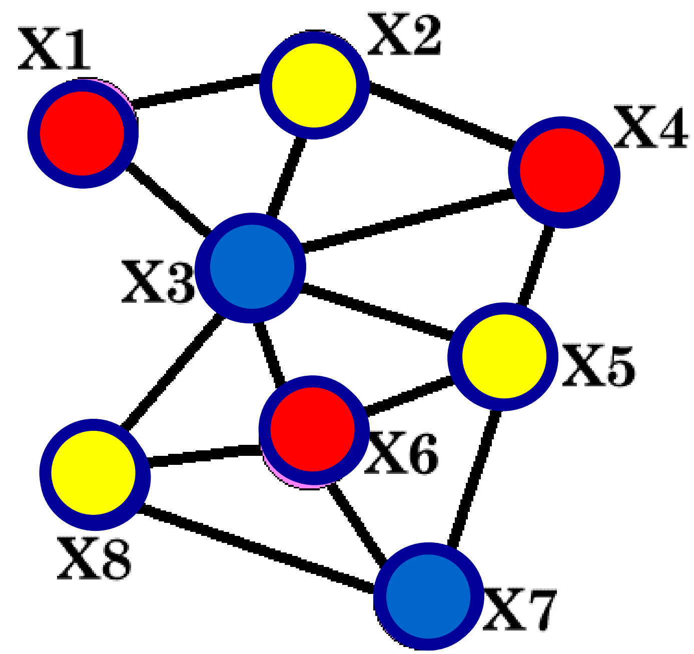




Map to be colored



Equivalent graph



Coloring the graph

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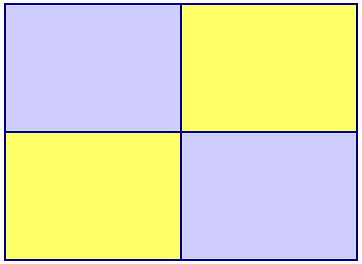
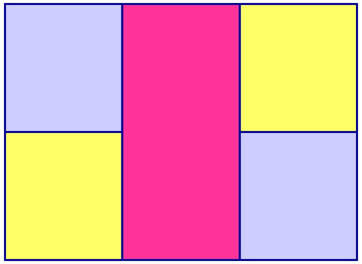
The Map Coloring Problem

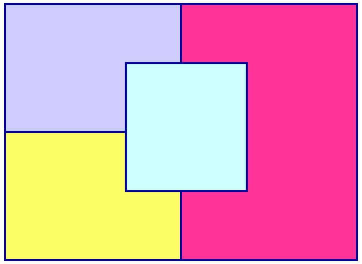
We want to color countries, oceans, lakes, and islands on a map so that no two adjacent areas have the same color.



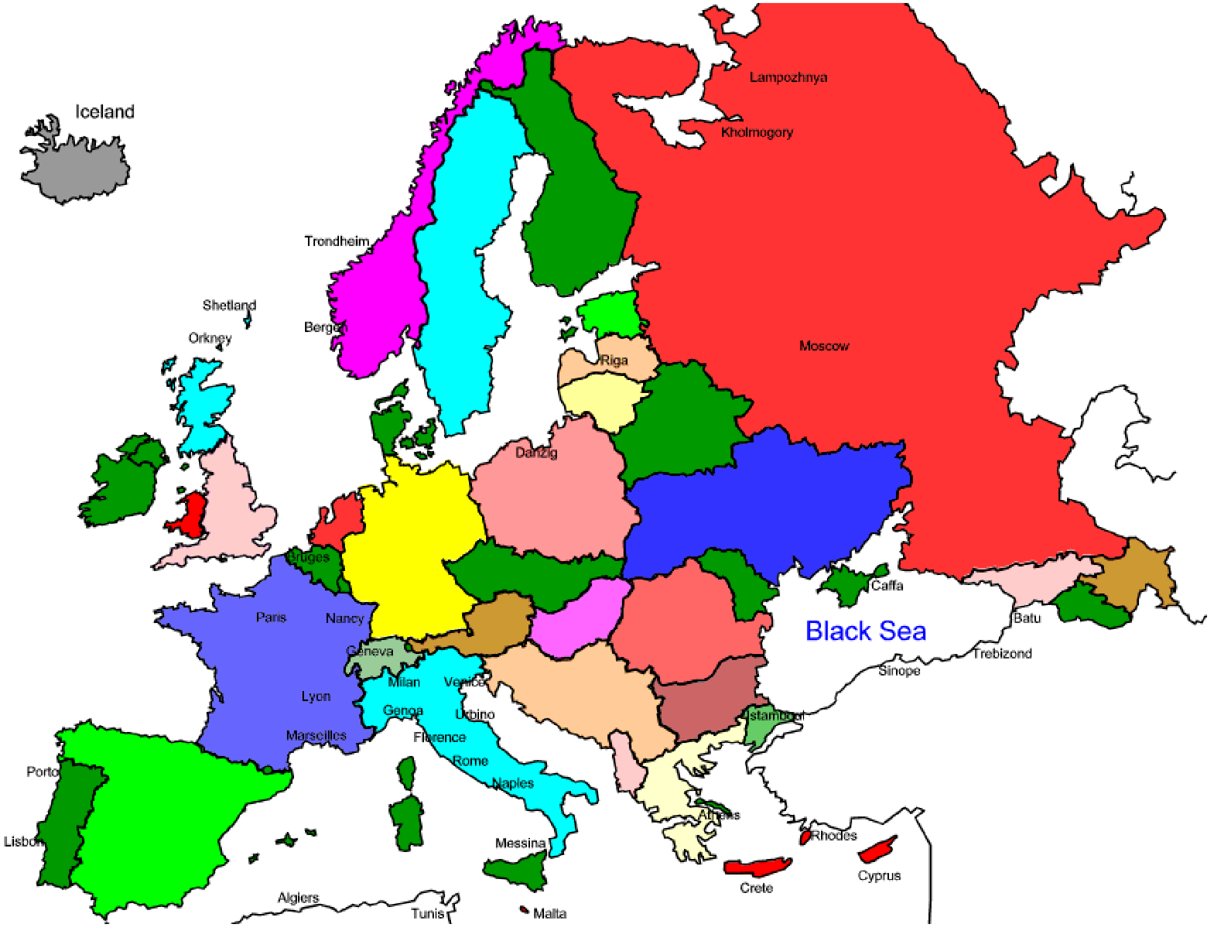
Two colors



Three colors



Four colors



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The Four Color Theorem

Can *all* map be coloured using only four colours?

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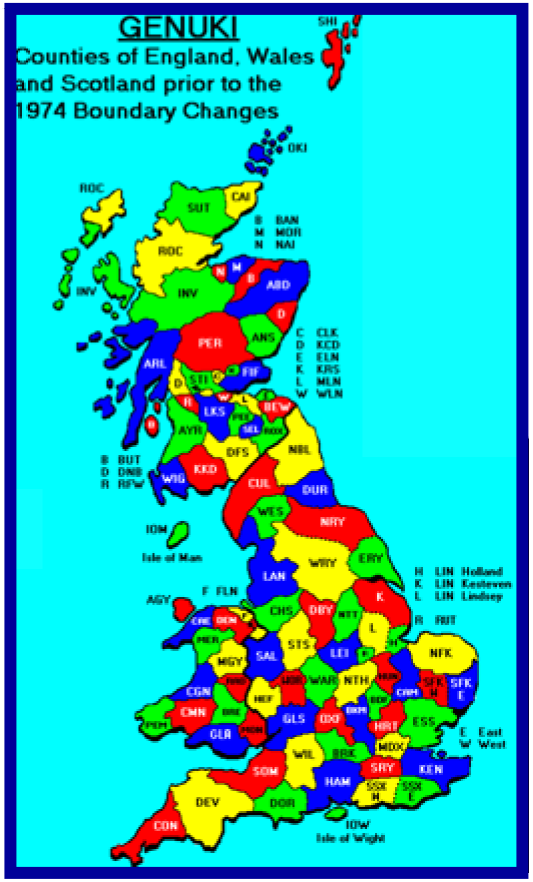
150 years of history…

- 1852 Conjecture (*Guthrie* → *DeMorgan*)
- 1878 Publication (*Cayley*)
- 1879 First proof (*Kempe*)
- 1880 Second proof (*Tait*)
- 1890 Rebuttal (*Heawood*)
- 1891 Second rebuttal (*Petersen*)
- 1913 Reducibility, connexity (*Birkhoff*)
- 1922 Up to 25 regions (*Franklin*)
- 1969 Discharging (*Heesch*)
- 1976 Computer proof (*Appel & Haken*) @UIUC
- 1995 Streamlining (*Robertson & al.*)
- 2005 COQ proof (*Gonthier*)



I took a *Combinatoric* course with Ken Appel in Fall 1979

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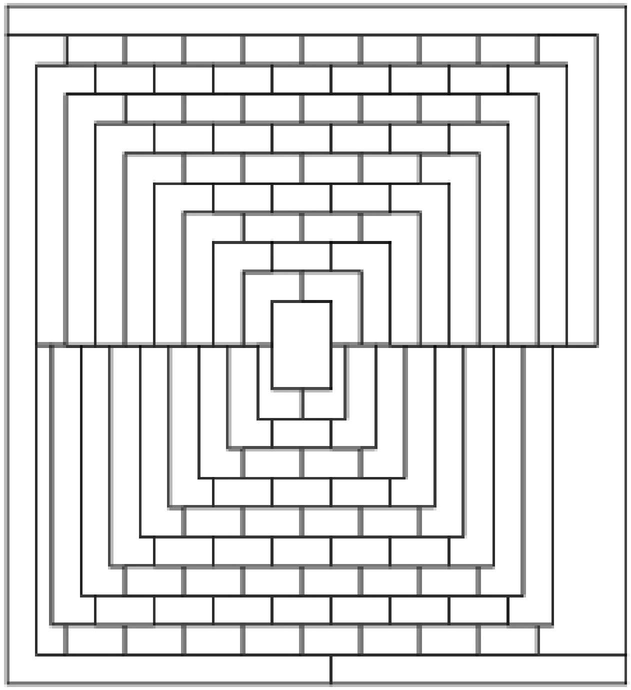
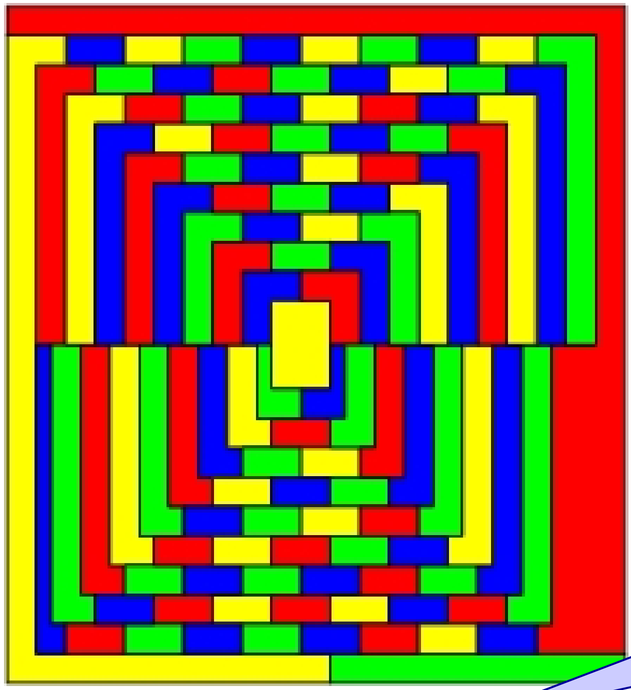
Does four colour suffices?





Martin Gardner published in Scientific American (*April* 1975) this map of 110 regions. He claimed that the map *requires five colors* and constitutes a counterexample to the four-color theorem.

However, the coloring of Wagon, obtained algorithmically using *Mathematica*, clearly shows that this map is, in fact, four-colorable.

Source: http://mathworld.wolfram.com/Four-ColorTheorem.html

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Activity Period #2:

Map Colouring &

Fish in a Tank

(10 minutes)

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Review of Hands-on Activity #2

- How many colours did the map need?
  - You should never need more than 4 colours
- Did you know about the "Four-Colour Theorem":
- How many fish tanks did you need?

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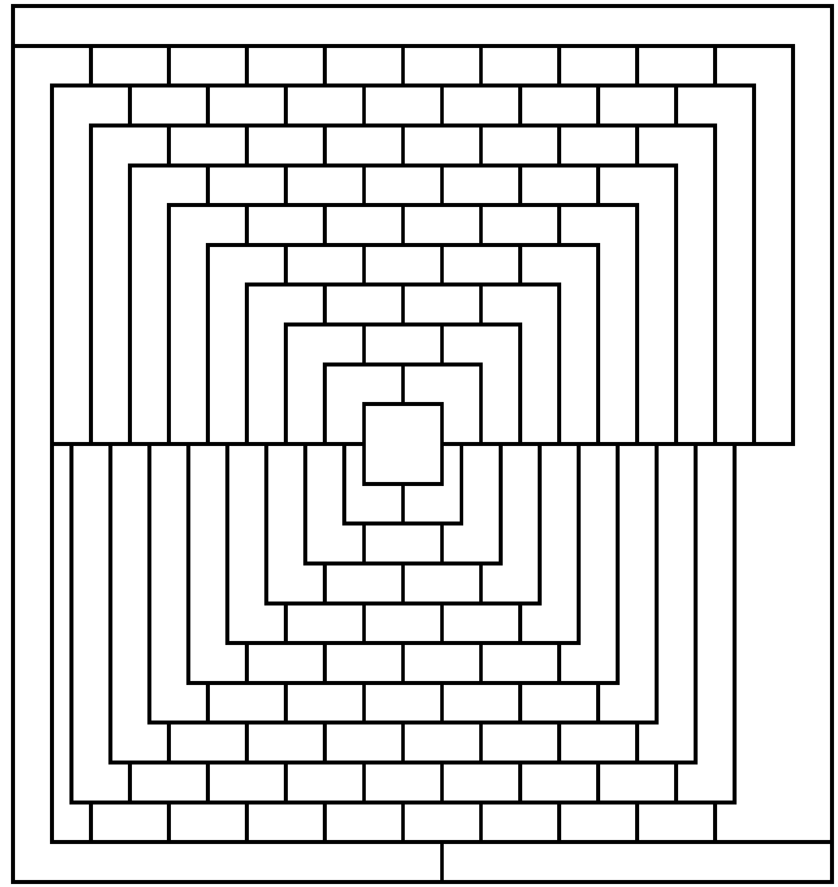
Activity 5: Color These Maps

Use as few colors as possible

Real map: One color already used



Made-up map

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Summary of Problem Modelling

	Tourist Problem	Fish in a tank	Frequency Assignment	Map Coloring
Nodes	*places*	*fishes*	*radio stations*	*Countries*
Edges / Conflicts	*tourist want to visit both places*	*cannot be placed in same tank*	*interference if placed too near*	*share a common border*
Colors	*bus trips to places*	*fish tanks*	*signal frequencies*	*color*
Others	*The tourists*	--		

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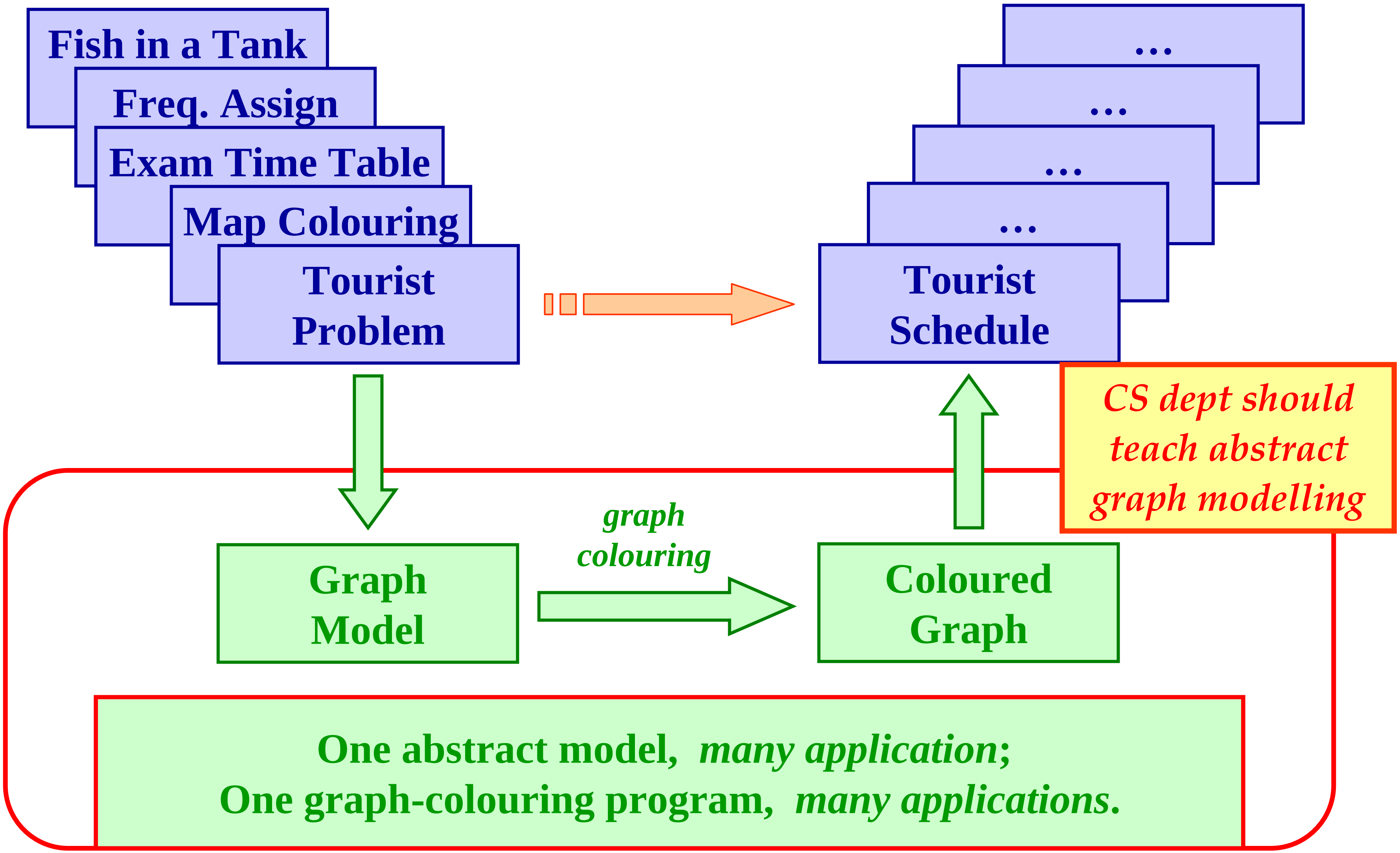
Why CS dept teach abstract problems?



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References…

On Graph Coloring and Applications:

1. http://www.geom.uiuc.edu/~zarembe/graph3.html
2. http://www.colorado.edu/education/DMP/activities/graph/ddghnd03.html
3. Lots of other links available

On the Four Color Theorem:

1. http://en.wikipedia.org/wiki/Four_color_theorem
2. http://www.maa.org/reviews/fourcolors.html
3. http://www.math.gatech.edu/~thomas/FC/fourcolor.html
4. http://www.mathpages.com/home/kmath266/kmath266.htm

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End of Talk on Tourist Problem!

School of Computing

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