"SOC InfoComm Camps"

Computational Problem Solving

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(19 December 2007)

Science Focus – Algorithmic PS

"SOC InfoComm Camps"

- Algorithmic Problem Solving
  - The Tourist Problem
  - Graph Colouring and Applications
  - Packing Problems and Applications

(includes some real-life problems)

Experience the fun of problem solving

The Tourist Problem

- Organization
  - The Tourist Problem
  - Analysis and Simplifications
  - Problem Modelling (with *Graphs*)
  - Solving the *Graph* Model
  - Mapping back the Solution
  - Moral of the Story

Experience the fun of problem solving

The Tourist Problem…

Given: A list of tourist, each with his/her list of places to visit.
To do: Schedule bus rides for them so that each tourist visits all the places in his/her list.

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG
Frances	BG, SZG, JB
Gary	CG, OR
Harry	JG, CG

The Tourist Problem (Entities)

- Good to know the entities we are dealing with…
  - The Tourists:
    T = { *A, B, C, D, E, F, G, H* }
  - The Attractions (Places):
    P = { *BG, CG, JB, JG, OR, SI, VC, SZG* }

… Places of Attraction …

Place	Common Name	Place	Common Name
BG	Botanical Gardens	CG	Chinese Gardens
JB	Jurong Birdpark	JG	Japanese Gardens
OR	Orchard Road	SI	Sentosa Island
SZG	Spore Zoological Gardens	VC	VivoCity

The Tourist Problem (Analysis…)

Some Simplifications: Consider
- Aaron { SZG, BG, JB }
- ~~Frances { SZG, BG, JB }~~

Also consider
- David { JG, CG, OR }
- ~~Gary { CG, OR }~~

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
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Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG
~~Frances~~	~~BG, SZG, JB~~
~~Gary~~	~~CG, OR~~
~~Harry~~	~~JG, CG~~

Simplification Rule:
If $P(T_1) \subseteq P(T_2)$, then tourist T_1 can just "follows" tourist T_2. Thus, we can omit T_1 from consideration.

Oh, can also omit Harry
- Betty { CG, JG, BG }
- ~~Harry { CG, JG }~~

The (Reduced) Tourist Problem…

Given: A list of tourist, each with his/her list of places to visit.
To do: Schedule bus rides for them so that each tourist visits all the places in his/her list.

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

$T = \{ A, B, C, D, E \}$

$P = \{ BG, CG, JB, JG, OR, SI, VC, SZG \}$

The Tourist Problem – v0

Given: A list of tourist, each with his/her list of places to visit.
To do: Schedule bus rides for them so that each tourist visits all the places in his/her list.

Solution: (Singapore 1-Day Tour)
Put all the tourists on one bus.
Visit all eight places in 1 day.

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

What's Good: It works! One bus, one-day.
What's Bad: Too rushed. NO time to see anything!

Not interesting!

The Tourist Problem – v0.5

Given: A list of tourist, each with his/her list of places to visit.
To do: Schedule bus rides for them so that each tourist visits all the places in his/her list, *and*
C1: Each tourist visits *at most one place a day*.

Simple Solution:
Schedule *one trip to every place every day*.

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

What's Good: It works! Finish in 3 days. (*minimum*!)
What's Bad: Wasteful! 24 bus trips.

Also, not so interesting!

The Tourist Problem – v0.8

Given: A list of tourist, each with his/her list of places to visit.
To do: Schedule bus rides for them so that each tourist visits all the places in his/her list,
C1: Each tourist visits *at most one place a day*, *and*
C2: There is *at most one bus trip to each place*

Simple Solution:
Schedule *one trip per day*, each to a *different* place.

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

What's Good: It works! 8 trips.
What's Bad: It takes 8 days!

But wait… Did you see something interesting?

The Tourist Problem – v1.0

Given: A list of tourist, each with his/her list of places to visit.
To do: Schedule bus rides for them so that each tourist visits all the places in his/her list,
C1: Each tourist visits *at most one place a day*,
C2: There is *at most one bus trip to each place*, *and*
C3: *minimize* the number of days to complete mission.

Observation:
On the same day,
cannot schedule SZG and BG
can schedule SZG and OR

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
Betty	CG, JG, BG
Cathy	VC, SI, OR
David	JG, CG, OR
Evans	CG, JG, SZG

How to model all these constraints?

The Graph Model

- **What is a graph?**
  - **eg:** $y = sin(bx)$

- **No. Not this type of graph.**

The Graph Model

- Graph $G = (V, E)$
  - V is a set of vertices, nodes (circles)
  - E is a set of edges (connections)

An Instance of Tourist Problem	
Tourist	Places of Interest
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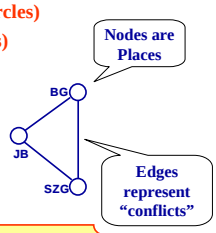
Evans	CG, JG, SZG



In our graph, nodes are places, and edges in the graph means conflicts.

Graph Model for the Tourist Problem

An Instance of Tourist Problem	
Tourist	Places of Interest
Aaron	SZG, BG, JB
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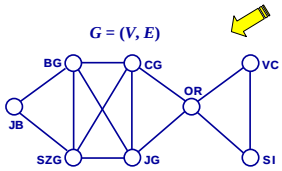
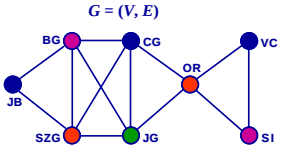
Evans	CG, JG, SZG



The graph $G = (V, E)$ captures all the conflicts for our tourist problem instance.

Graph Model for the Tourist Problem

- What's good about the graph model?
  - *very simple* !
  - easy to spot conflicts and *non-conflicts*

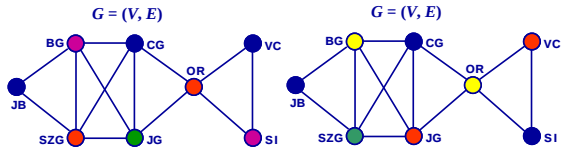


On Day 1, can schedule SZG, OR [Any more? Why?]
On Day 2, can schedule JB, CG, VC
On Day 3, can schedule BG, SI
On Day 4, can schedule JG

Graph Coloring Problem

- Given a graph $G = (V, E)$, colour the vertices in V so that any two vertices that are connected by an edge in E will have *different* colors.

  We want to *minimize* the number of colors.



Number of colors used to color the graph G = *Number of days needed to complete the schedule*

Activity Period #1:

Graph Colouring Exercises

(10 minutes)

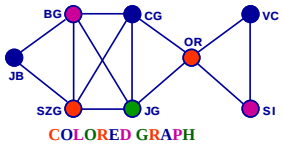
Review of Activity

- Is Graph Colouring fun?
  - Did you *really* used different colours?
- How many colours was did you use?
  - Q1 and Q2
- What about the cycles (Q3):
  - Q3(a) C_6 (a cycle of length 6)?
  - C_6 (a cycle of length 6)?
- What about the final graph?

Get Solution to Tourist Problem - 1

❑ Colored graph ⇒ "Bus Schedule"

$G = (V, E)$



COLORED GRAPH

Color	Day	Place
	1	SZG, OR
	2	JB, CG, VC
	3	BG, SI
	4	JG

1. What about the list of tourists on each bus?
   Can we get it from the graph model?
   NO. *Why* NOT.

The Tourist Problem…

An Instance of Tourist Problem

Tourist	Places of Interest
Aaron	SZG, BG, JB
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Frances	BG, SZG, JB
Gary	CG, OR
Harry	JG, CG

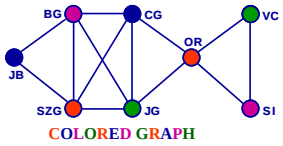
Alternative Representation

Tourist	BG	CG	JB	JG	OR	SI	SZG	VC
Aaron	X		X				X	
Betty	X	X		X				
Cathy					X	X		X
David		X		X	X			
Evans		X		X			X	
Frances	X		X				X	
Gary		X			X			
Harry		X		X				

Get Solutions to Tourist Problem (2)

❑ Colored graph ⇒ "Bus Schedule"

$G = (V, E)$



COLORED GRAPH

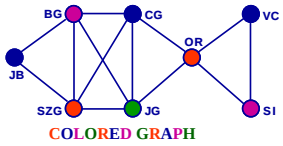
Color	Day	Place
	1	SZG, OR
	2	JB, CG, ~~VC~~
	3	BG, SI
	4	JG, VC

1. What about the list of tourists on each bus?
2. What if you only have 2 buses?
   • can color vertex VC green.

Get Solutions to Tourist Problem (3)

❑ Colored graph ⇒ "Bus Schedule"

$G = (V, E)$

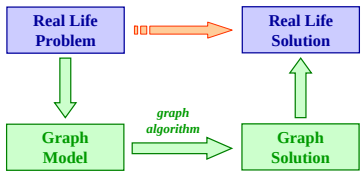


COLORED GRAPH

Color	Day	Place
	1	SZG, OR
	2	JB, CG, VC
	3	BG, SI
	4	JG

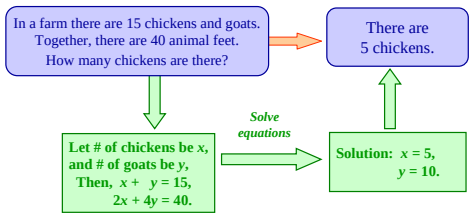
1. What about the list of tourists on each bus?
2. What if you only have 2 buses?
3. Can we re-order the colors?
4. Can we use fewer colors (fewer days)?

Graph Modelling…



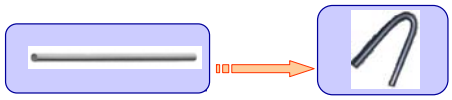
Modelling…

❑ Nothing new. You do it *all* the time.



Modelling: Another example

❑Bend a steel bar

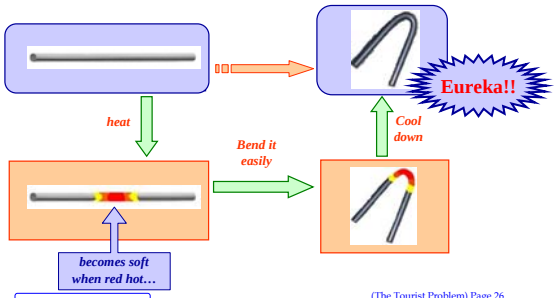


(Direct method)

Man bending steel rod

Modelling: Another example (2)

❑Bend a steel bar (using *transformation*)



Modelling in Tourist Problem

Recap: Our Graph modelling…

Graph Model	Tourist Problem			
Nodes	*places*			
Edges / Conflicts	*tourist want to visit both places*			
Colors	*bus trips to places*			
Others	*The tourists*			

Moral of the Story

- **The Tourist Problem:**
  - Some problems are EASY. (don't complicate them)
  - Get a *simple* solution first.
    then *analyze* it, *improve* it, *refine* it.
  - Solution depend on the questions asked
  - It is important to ASK QUESTIONS.
  - Theoretical modelling and analysis are beneficial
- **Modelling**
  - Abstract modeling simplifies problem and solution!
  - Abstract model is transferable.
  - Models don't answer everything.

End of the

Tourist Problem!

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Graph Coloring & Applications

- **Where is Graph Coloring used**
  - The Tourist Problem [done]
  - Map Coloring
  - Fish in a Tank
  - Frequency assignment in wireless networks
  - Time Table Scheduling
  - And a whole lot more…

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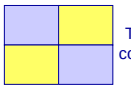
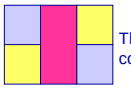
Experience the fun of problem solving

The Map Coloring Problem

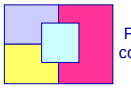
We want to color countries, oceans, lakes, and islands on a map so that no two adjacent areas have the same color.



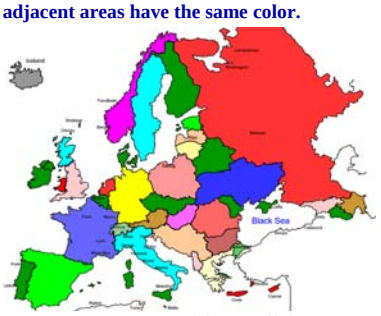
Two colors



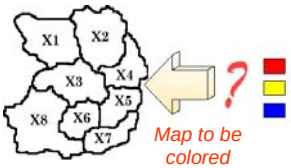
Three colors



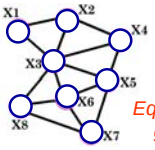
Four colors



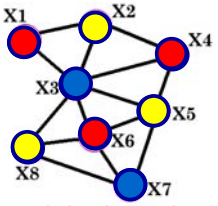
Map and Graph Coloring



Map to be colored



Equivalent graph

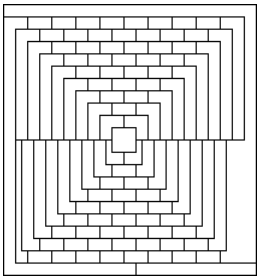


Coloring the graph

Activity 5: Color These Maps

Use as few colors as possible

Real map: One color already used



Made-up map

Activity Period #2:

Map Colouring &
Fish in a Tank
(10 minutes)

Review of Hands-on Activity #2

- **How many colours did the map need?**
  - **You should never need more than 4 colours**
- **Did you know about the "Four-Colour Theorem":**
- **How many fish tanks did you need?**

Summary of Problem Modelling

	Tourist Problem	Map Coloring	Fish in a tank	
Nodes	*places*	*Countries*	*fishes*	
Edges / Conflicts	*tourist want to visit both places*	*share a common border*	*cannot be placed in same tank*	
Colors	*bus trips to places*	*color*	*fish tanks*	
Others	*The tourists*		--	

Thank you!

References…

1. http://www.geom.uiuc.edu/~zarembe/graph3.html
2. http://www.colorado.edu/education/DMP/activities/graph/ddghnd03.html
3.

Summary of Problem Modelling

	Tourist Problem	Fish in a tank	Frequency Assignment	Map Coloring
Nodes	*places*	*fishes*	*radio stations*	*Countries*
Edges / Conflicts	*tourist want to visit both places*	*cannot be placed in same tank*	*interference if placed too near*	*share a common border*
Colors	*bus trips to places*	*fish tanks*	*signal frequencies*	*color*
Others	*The tourists*	--		