created by Syed Abdullah

MA1521: Annexes

Rules of Differentiation

$f(x)$	$f'(x)$
$(f(x))^n$	$n(f'(x))(f(x))^{(n-1)}$
$\sin(f(x))$	$f'(x)(\cos(f(x)))$
$\cos(f(x))$	$-f'(x)(\sin(f(x)))$
$\tan(f(x))$	$f'(x)\sec^2(f(x))$
$\csc(f(x))$	$-f'(x)\csc(f(x))\cot(f(x))$
$\sec(f(x))$	$f'(x)\sec(f(x))\tan(f(x))$
$\cot(f(x))$	$-f'(x)\csc^2(f(x))$
$e^{f(x)}$	$f'(x)e^{f(x)}$
$\ln(f(x))$	$\frac{f'(x)}{f(x)}$
$\sin^{-1}(f(x))$	$\frac{f'(x)}{\sqrt{1-(f(x))^2}}$
$\cos^{-1}(f(x))$	$\frac{-f'(x)}{\sqrt{1-(f(x))^2}}$
$\tan^{-1}(f(x))$	$\frac{f'(x)}{1+(f(x))^2}$

Chain Rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

Product Rule:

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{(g(x))^2}$$

Rules of Integration

ALL integration includes arbitrary **constant**!

$f(x)$	$\int$
$(ax+b)^n$	$\frac{(ax+b)^{n+1}}{a(n+1)}$
$\frac{1}{ax+b}$	$\frac{\ln\lvert ax+b\rvert}{a}$
e^{ax+b}	$\frac{e^{ax+b}}{a}$
$\cos(ax+b)$	$\frac{\sin(ax+b)}{a}$
$\sin(ax+b)$	$-\frac{\cos(ax+b)}{a}$
$\tan(ax+b)$	$\frac{\ln\lvert\sec(ax+b)\rvert}{a}$
$\sec(ax+b)$	$\frac{\ln\lvert\sec(ax+b)+\tan(ax+b)\rvert}{a}$
$\csc(ax+b)$	$-\frac{\ln\lvert\csc(ax+b)+\cot(ax+b)\rvert}{a}$
$\cot(ax+b)$	$\frac{\ln\lvert\sin(ax+b)\rvert}{a}$
$\sec^2(ax+b)$	$\frac{\tan(ax+b)}{a}$
$\csc^2(ax+b)$	$-\frac{\cot(ax+b)}{a}$
$\sec(ax+b)\tan(ax+b)$	$\frac{\sec(ax+b)}{a}$
$\csc(ax+b)\cot(ax+b)$	$-\frac{\csc(ax+b)}{a}$
$\frac{1}{a^2+(x+b)^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x+b}{a}\right)$
$\frac{1}{\sqrt{a^2-(x+b)^2}}$	$\sin^{-1}\left(\frac{x+b}{a}\right)$
$\frac{-1}{\sqrt{a^2-(x+b)^2}}$	$\cos^{-1}\left(\frac{x+b}{a}\right)$
$\frac{1}{a^2-(x+b)^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x+b+a}{x+b-a}\right\rvert$
$\frac{1}{(x+b)^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x+b-a}{x+b+a}\right\rvert$
$\frac{1}{\sqrt{(x+b)^2+a^2}}$	$\ln\left\lvert(x+b)+\sqrt{(x+b)^2+a^2}\right\rvert$
$\frac{1}{\sqrt{(x+b)^2-a^2}}$	$\ln\left\lvert(x+b)+\sqrt{(x+b)^2-a^2}\right\rvert$

Completing the Squares For integrals involving fractions with quadratic equations:

$$ax^2+bx+c = a\left(x+\frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

Trigonometric Identities Use trigo identities to simplify complex trigonometrical equations

Partial Fractions If fraction is in the form of a partial fraction, use p.f. to split it into multiple terms

Change of Variable (Substitution)

For integrals in the form: $\int f(g(x))g'(x)dx$

Formula

$$\int f(g(x))g'(x)dx = \int f(u)du$$

By Parts

For integrals in the form: $\int (f'(x))g(x)dx$ & $\ln(x)$ integrations

Formula

$$\int (f'(x))g(x)dx = f(x)g(x) - \int f(x)(g'(x))dx$$

Priority for $g(x)$: LIATE

Logarithmic, **I**nverse trigonometric, **A**lgebraic, **T**rigo, **E**xponential ($e^x, 19^x$)

Convergence Tests

Test	Conditions [Concl]	Type of u_n
nth term $\lim_{n\to\infty} u_n$	$L \neq 0$ [Divergent]	$\lim_{n\to\infty} \neq 0$
	$L = 0$ [Inconclusive]	
p-series	Converges iff $p > 1$	$\sum_n^\infty \frac{1}{n^p}$
	Diverges iff $p \leq 1$	
Ratio $\lim_{n\to\infty} \left(\frac{u_{n+1}}{u_n}\right) = L$	$L \in [0, 1)$ [Conv]	Factorials,
	$L \in (1, \infty)$ [Div]	Polynomials
	$L = 1$ [Inconcl.]	Exponentials
Root $\lim_{n\to\infty} u_n^{1/n} = L$	$L \in [0, 1)$ [Conv]	Polynomials
	$L \in (1, \infty)$ [Div]	Exponentials
	$L = 1$ [Inconcl.]	
Comparison Test	b_n conv $\rightarrow a_n$ conv	$a_n \leq b_n$
	a_n div $\rightarrow b_n$ div	
Limit Comparison $\lim_{n\to\infty} \frac{y_n}{x_n} = L$	$L \in (0, \infty)$ [y_n conv iff x_n conv]	Frac types (dominating powers)
	$L = 0$ [x_n conv $\rightarrow y_n$ conv]	
	$L = \infty$ [x_n div $\rightarrow y_n$ div]	
Alternating Series	Convergent if: $u_{n+1} < u_n$, $\lim_{n\to\infty} u_n = 0$	$\sum_n^\infty (-1)^n u_n$
Geometric Series	Converges iff $\lvert r \rvert < 1$	$\sum_{n=k}^\infty r^n$

Basic Transformation of Graphs

Equation	Transformation
$f(x-k)$	translate k units to the **right**
$f(x+k)$	translate k units to the **left**
$f(x)+k$	translate k units **upwards**
$f(x)-k$	translate k units **downwards**
$f(-x)$	reflection in y-axis
$-f(x)$	reflection in x-axis
$kf(x)$	scale along y-axis by k times
$f(kx)$	scale along x-axis by $1/k$

Graphs of Common Functions