CS3245

Information Retrieval

7

Lecture 7: Scoring, Term Weighting and the Vector Space Model

Last Time: Index Compression

- Collection and vocabulary statistics: Heaps' and Zipf's laws
- Dictionary compression for Boolean indexes
  - Dictionary string, blocks, front coding
- Postings compression:
  - Gap encoding and variable byte encoding

Data structure	Size in MB
dictionary, fixed-width	11.2
dictionary, term pointers into string	7.6
with blocking, k = 4	7.1
with blocking & front coding	**5.9**
postings, uncompressed (32-bit words)	400.0
postings, variable byte encoded	**116.0**

Today: Ranked Retrieval

- Scoring documents
- Term frequency
- Collection statistics
- Weighting schemes
- Vector space scoring

- Parametric and zone indexes (Section 6.1) will be covered next week.

Recap on Boolean Retrieval

- Thus far, our queries have all been Boolean.
  - Documents either match or don't.
- **Good** for expert users with precise understanding of their needs and the collection.
  - Also good for applications: Applications can easily consume 1000s of results.
- **Not good** for the majority of users.
  - Most users don't want to wade through 1000s of results.
    - This is particularly true of web search.
  - Most users may have difficulty in writing Boolean queries.

Problem with Boolean search: Feast or Famine

Boolean queries often result in either too few (=0) or too many (1000s) results.

- Q1: “Windows 10 login” AND KB3081444 → 0 hits
- Q2: “Windows 10 login” OR KB3081444 → 377M hits
  - Also called “information overload”

It takes a lot of skill to come up with a query that produces a manageable number of hits.

  - AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in **ranked retrieval models**, the system returns an ordering over the (top) documents in the collection with respect to a query
- **Free text queries**: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- Two separate choices, but in practice, ranked retrieval models are associated with free text queries

Ranked retrieval

When a system produces a ranked result set, large result sets are not an issue

- We just show the top k (≈ 10) results
- We don't overwhelm the user

Premise: the ranking algorithm works

Scoring as the basis of ranked retrieval

We wish to return in order the documents most likely to be useful to the searcher.

How can we rank the documents in the collection with respect to a query?

Assign a score – say in [0, 1] – to each document

This score measures how well document and query "match".

Take 1: Jaccard coefficient

- Recall the Jaccard coefficient from Chapter 3 (spelling correction): a measure of overlap of two sets A and B

Jaccard $(A,B) = |A \cap B| \,/\, |A \cup B|$
Jaccard $(A,A) = 1$
Jaccard $(A,B) = 0$ if $A \cap B = 0$

Pros:

- A and B don't have to be the same size.
- Always assigns a number between 0 and 1.

Blanks on slides, you may want to fill in

Jaccard coefficient: Scoring example

What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?

- Query: *ides of march*
- Document 1: *caesar died in march*
- Document 2: *the long march*

Issues with Jaccard for scoring

- It doesn't consider ***term frequency*** (how many times a term occurs in a document)
- ***Rare terms*** in a collection are more informative than frequent terms. Jaccard doesn't consider this information either.

Let's start with a one-term query…

- If the query term does not occur in the document, the score should be 0
- The more frequent the query term in the document, the higher the score (should be)

Recap: Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a **binary vector** $\in \{0,1\}^{|V|}$

1. Term-document count matrices

- Store the number of occurrences of a term in a document:
  - Each document is a **count vector** in $\mathbb{N}^{v}$: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Term frequency tf

- The term frequency $tf_{t,d}$ of term t in document d is defined as the number of times that t occurs in d.
- We want to use *tf* when computing query-document match scores. But how?
- Raw term frequency is not what we want:
  - Relevance does not increase proportionally with term frequency.
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence. But not 10 times more relevant.

Note: frequency = count in IR

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

e.g. 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.

- Score for a document-query pair: sum over terms t in both q and d:

$$\text{score} = \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$$

- The score is 0 if none of the query terms is present in the document.

2. Document frequency

- Rare terms are more informative than frequent terms
  - Recall stop words
  - Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
  - We want a high weight for rare terms like *arachnocentric*.

Blanks on slides, you may want to fill in

Document frequency, continued

- **Frequent terms** are less informative than **rare terms**
- Consider a query term that is frequent in the collection (e.g., *high, increase, line*)
- A document containing such a term is more likely to be relevant than a document that doesn't ...
- For frequent terms, we want high positive weights for words like *high, increase,* and *line* ...
- We will use document frequency (df) to capture this.

idf weight

- df_t is the <u>document</u> frequency of t: the number of documents that contain t
  - df_t is an inverse measure of the informativeness of t
  - $df_t \leq N$
- We define the idf (inverse document frequency) of t by

$$\mathrm{idf}_t = \log_{10}\left(N/\mathrm{df}_t\right)$$

  - We use log (N/df$_t$) instead of N/df$_t$ to "dampen" the effect of idf.

Example: suppose *N* = 1 million

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$\text{idf}_t = \log_{10}(N/\text{df}_t)$$

There is one idf value for each term *t* in a collection.

Blanks on slides, you may want to fill in

Effect of *idf* on ranking

- Does idf have an effect on ranking for one-term queries, like iPhone?
- *idf* has on ranking one term queries
  - idf affects the ranking of documents for queries with at least
  - For the query **capricious person**, idf weighting makes occurrences of **capricious** count for much more in the final document ranking than occurrences of **person**.

Collection vs. Document frequency

- The collection frequency of *t* is the number of occurrences of *t* in the collection, counting multiple occurrences.
- Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

- Which word is a better search term (and should get a higher weight)?

tf-idf weighting

- The *tf-idf* weight of a term is the product of its *tf* weight and its *idf* weight.

$$w_{t,d} = (1 + \log \mathrm{tf}_{t,d}) \times \log_{10}(N / \mathrm{df}_t)$$

- **Best known weighting scheme IR**
  - Note: the "-" in *tf-idf* is a hyphen, not a minus sign!
  - Alternative names: *tf.idf, tf x idf*
- **Increases** with the number of occurrences within a document
- **Increases** with the rarity of the term in the collection

Final ranking of documents for a query

$$\text{Score}(q,d) = \sum_{t \in q \cap d} \text{tf.idf}_{t,d}$$

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Documents as vectors

- So we have a $|V|$-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- High-dimensional: tens of thousands of dimensions; each dictionary term is a dimension
- These are very sparse vectors - most entries are zero.

Bag of words model

- Con: Vector representation doesn't consider the ordering of words in a document

  Moonlight bests La La Land at the Oscars and *La La Land bests Moonlight at the Oscars* have the same vectors

- In a sense, this is a step back: The positional index was able to distinguish these two documents.
- We will look at "recovering" positional information later in this course.

Queries as vectors

- Key idea 1: Do the same for queries: represent them as vectors in the space; they are "mini-documents"
- Key idea 2: Rank documents according to their proximity to the query in this space

- proximity = similarity of vectors
- proximity ≈ inverse of distance

Motivation: Want to get away from the you're-either-in-or-out Boolean model.

Instead: rank more relevant documents higher than less relevant documents

Blanks on slides, you may want to fill in

Formalizing vector space proximity

- First cut: distance between two points
  - (= distance between the end points of the two vectors)
- Euclidean distance?

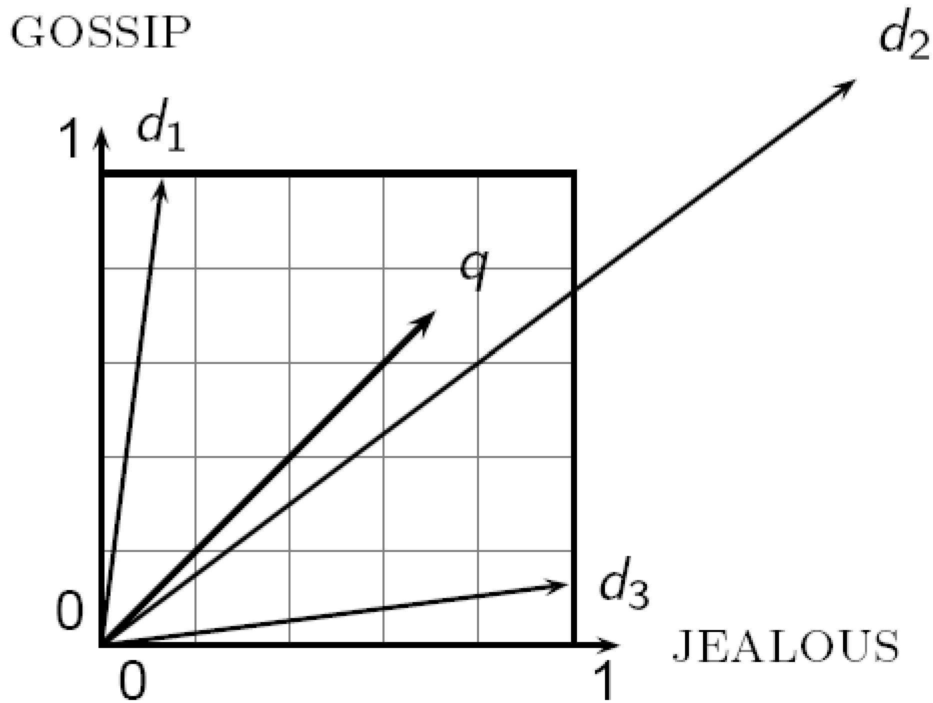
$$d(\mathbf{p},\mathbf{q}) = d(\mathbf{q},\mathbf{p}) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + \cdots + (q_n - p_n)^2}$$

$$= \sqrt{\sum_{i=1}^{n} (q_i - p_i)^2}.$$

- Euclidean distance is a bad idea ...

Why distance is a bad idea

- The Euclidean distance between $\vec{q}$ and $\overrightarrow{d_2}$ is large even though the distribution of terms in the query $\vec{q}$ and the distribution of terms in the document $\overrightarrow{d_2}$ are very similar.



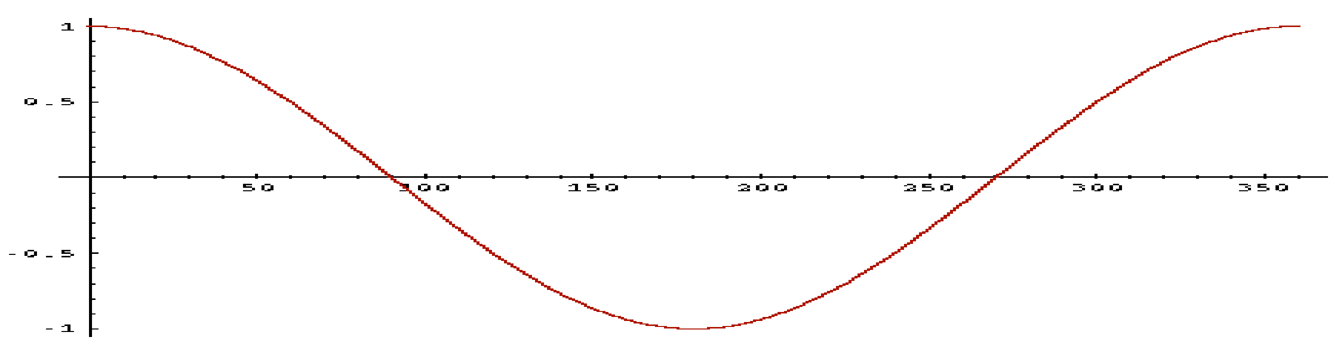
Use angle instead of distance

- Distance counterexample: take a document $\vec{d}$ and append it to itself. Call this document $\overrightarrow{d'}$.
- "Semantically" $\vec{d}$ and $\overrightarrow{d'}$ have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.

- Key idea: Rank documents according to the angle with query.

From angles to cosines

- The following two notions are equivalent.
  - Rank documents in decreasing order of the angle between query and document
  - Rank documents in increasing order of cosine(query, document)
- Cosine is a monotonically decreasing function for the interval [0°, 180°]



cosine (query, document)

$$\vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i = |\vec{q}|\,|\vec{d}| \cos(\vec{q}, \vec{d})$$

$$\cos(\vec{q}, \vec{d}) = \frac{\vec{q} \bullet \vec{d}}{|\vec{q}||\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\ \sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term i in the query

d_i is the tf-idf weight of term i in the document

$\cos(\vec{q}, \vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$ … or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

Length normalization

- The vectors in the computation of cosine similarity are in fact *length normalized* by dividing each of its components by its length:

$$\left|\vec{x}\right| = \sqrt{\sum_i x_i^2}$$

- Such normalization makes the weights comparable across different vectors despite their original lengths.
- Effect on the two documents $\vec{d}$ and $\overrightarrow{d'}$ (d appended to itself) from the earlier slide: they have identical vectors after length normalization.

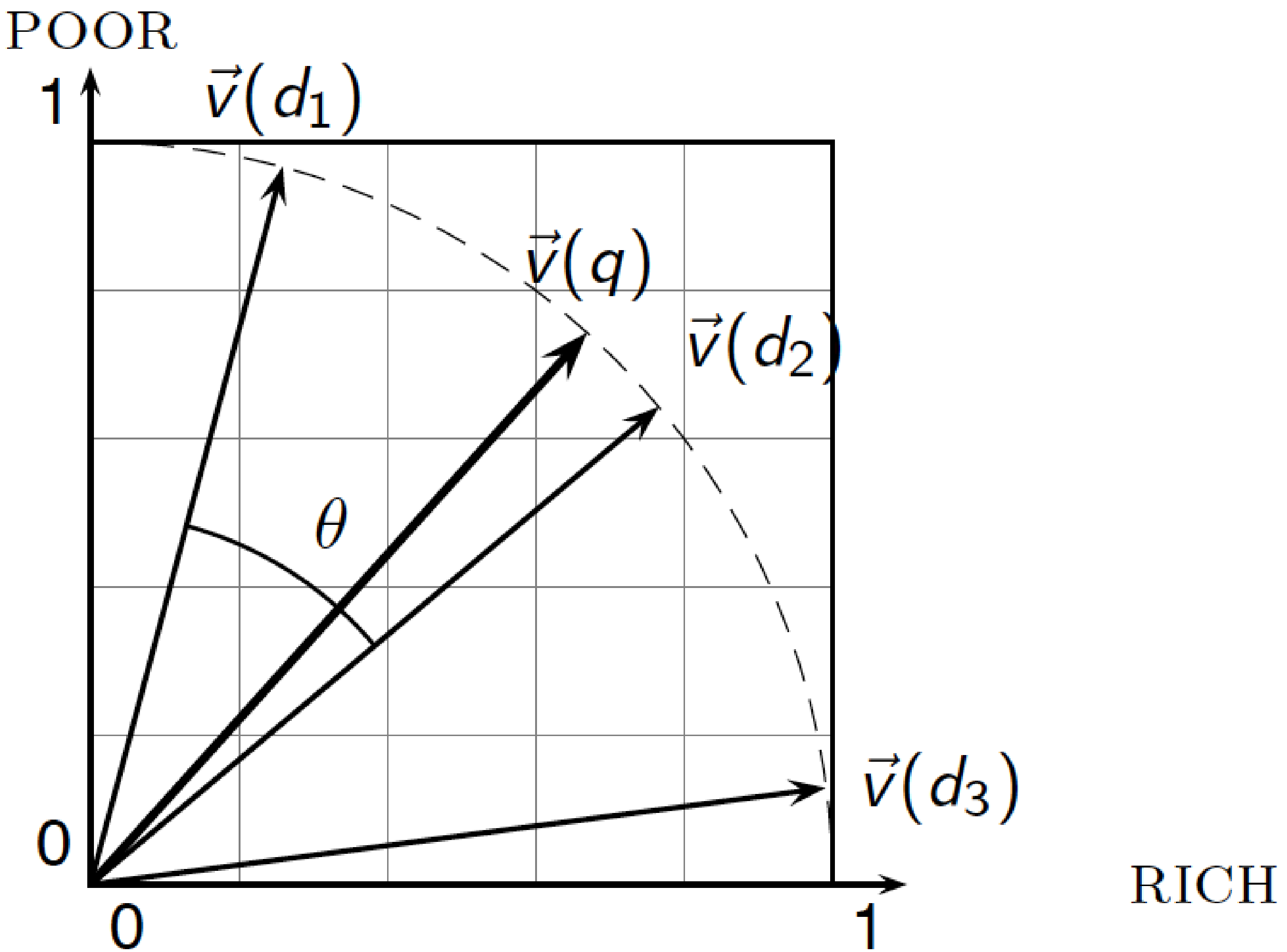
Cosine for length-normalized vectors

- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q},\vec{d}) = \vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

for length normalized $\vec{q}$ and $\vec{d}$

Cosine similarity illustrated



Cosine similarity among 3 documents

How similar are the novels
SaS: *Sense and Sensibility*
PaP: *Pride and Prejudice*, and
WH: *Wuthering Heights*?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we do not do idf weighting.

3 document example (cont'd)

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) $\approx 0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0 \approx$ 0.94

cos(SaS,WH) $\approx$ 0.79

cos(PaP,WH) $\approx$ 0.69

Computing cosine scores

CosineScore(q)

float $Scores[N] = 0$
float $Length[N]$
for each query term t
do calculate $w_{t,q}$ and fetch postings list for t
    for each pair(d, $tf_{t,d}$) in postings list
    do $Scores[d] += w_{t,d} \times w_{t,q}$
Read the array *Length*
for each d
do $Scores[d] = Scores[d]/Length[d]$
return Top K components of $Scores[]$

tf–*idf* weighting has many variants

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \ldots + w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N - \mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/CharLength^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1 + \log(\mathrm{tf}_{t,d})}{1 + \log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denote combination used with the notation *ddd.qqq*, using the acronyms from the table on the previous slide
- A very standard weighting scheme is lnc.ltc
  - Document: logarithmic *tf* (l as first character), no idf, cosine normalization

    A bad idea?
  - Query: logarithmic *tf* (l in the leftmost column), idf (t in the second column) and cosine normalization

$tf\text{–}idf$ example: lnc.ltc

Document: *car insurance auto insurance*
Query: *best car insurance*

Term	Document				Query						Prod
	tf-raw	tf-wt	wt	n'lize	tf-raw	tf-wt	df	idf	wt	n'lize	
auto	1	1	1	0.52	0	0	5000	2.3	0	0	0
best	0	0	0	0	1	1	50000	1.3	1.3	0.34	0
car	1	1	1	0.52	1	1	10000	2.0	2.0	0.52	0.27
insurance	2	1.3	1.3	0.68	1	1	1000	3.0	3.0	0.78	0.53

Quick Question: what is *N*, the number of docs?

Doc length $=\sqrt{1^2+0^2+1^2+1.3^2}\approx 1.92$

Score = 0+0+0.27+0.53 = 0.8

Summary and algorithm: Vector space ranking

1. Represent the query as a weighted *tf-idf* vector
2. Represent each document as a weighted *tf-idf* vector
3. Compute the cosine similarity score for the query vector and each document vector
4. Rank documents with respect to the query by score
5. Return the top *K* (e.g., *K* = 10) to the user

Resources for today's lecture

- IIR 6.2 – 6.4.3